Z-SCORES

- A value that shows how many Standard Deviations a data value is from the mean.
- When the **z-score is positive**, the data value is above the mean.
- When the **z-score is negative**, the data value is below the mean.
- When **z = 0**, the data value equal to the mean

Z-Score Formula

To find the z-score for a data value in a set that is normally distributed, use:

$$\boxed{z = \frac{X - \mu}{\sigma}}$$

X = the data value

μ = the mean average

σ = the standard deviation

EXAMPLES

starter: 1-3

$z = \frac{x - \mu}{\sigma}$

1. A radar detector records the speeds of a group of cars that pass by. If the mean is 46 mph and the standard deviation is 2.8 mph, find the z-scores for each data value. Use your z-table to find the percentile.

a) $\frac{52-46}{2.8} = 2.14$ 98.38%

b) $\frac{42-46}{2.8} = -1.43$ .0764 7.64%

c) $\frac{47-46}{2.8} = .36$ 64.06%

3. The mean number of total miles ran last week by each member on the track team was 4 with a standard deviation of 1.2. If Clay's z-score was 2.5, how many miles did he run?

$1.2 \cdot 2.5 = \frac{x-4}{\cancel{1.2}}$

$3 = x - 4$

$+4 \qquad +4$

$x = 7$

7 miles

4. A set test papers is normally distributed with a standard deviation of 5. If Riley scored an 83 with a z-score of -0.4, what was the mean?

$5 \cdot -.4 = \frac{83-\mu}{\cancel{5}}$

$-2 = 83 - \mu + 2$

$+\mu \qquad +\mu$

$\mu = 85$

the average was 85

5. The heights of a group of trees is normally distributed with a mean of 14.3 feet. If the z-score for a 20-foot tall tree is 1.9, what is the standard deviation?

$\sigma \cdot 1.9 = \frac{20-14.3}{\cancel{\sigma}}$

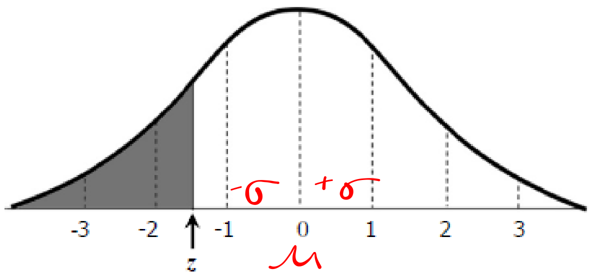
$\frac{\sigma \cdot 1.9}{1.9} = \frac{5.7}{1.9}$

$\sigma = 3$

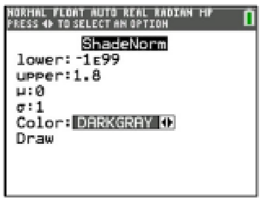
Stand. Dev. = 3

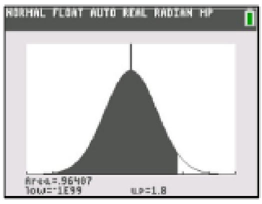
Standard Normal Distribution

- A normal distribution in which μ = 0 and σ = 1.
- The distribution is "standardized" by the z - score.



Calculator Directions





First, change your window dimensions:
Xmin: -4, Xmax: 4, Ymin: 0, Ymax: 0.5

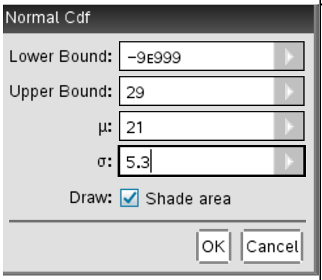
TI-84

To graph the curve and find the probability:

- **Step 1:** Find the *z*-score for each data value.
- **Step 2:** Hit 2nd -> VARS
- **Step 3:** Arrow over to **DRAW**
- **Step 4:** Select **1:ShadeNorm(**
- **Step 5:** Enter the **lower and upper bounds** for the *z*-scores
- **Step 6:** Scroll down and hit **DRAW**

Calculate it N Spire
*in calc screen
menu
6-statistics
5- Distributions
2-normalcdf
"cummulative distribution"

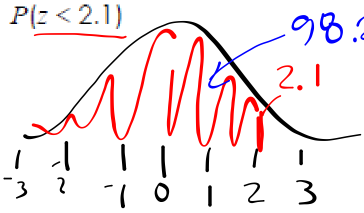
Graph it
home, lists
menu-4-2-2



Example: The golf scores in a tournament is normally distributed with a mean of 76 and a standard deviation of 2.6. Find each probability.

6. $P(z < 2.1)$

98.21%



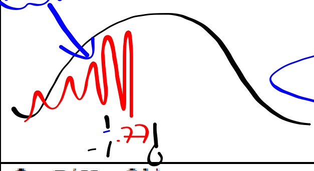
7. $P(X < 74)$

convert

$z = \frac{74-76}{2.6} = -.769$

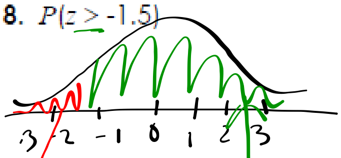
$\sim -.77$

22.06%



22.06%

8. $P(z > -1.5)$

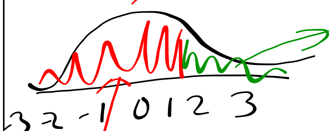


6.68%

100
− 6.68
93.32%

9. $P(X > 81)$

→ z = 1.92

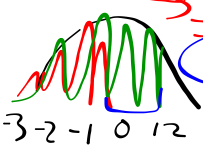


97.26%

100
− 97.26
2.74%

10. $P(-0.2 < z < 1.8)$

42.07%   96.41 %
− 42.07
54.34%



11. $P(77 < X < 83)$

z = .38   z = 2.69
64.80%   99.64%
− 64.80
34.8%

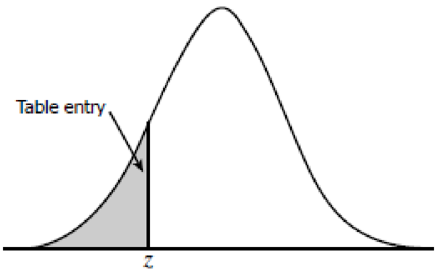


Table entry for z is the area under the standard normal curve to the left of z.

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
−3.4	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0003	.0002
−3.3	.0005	.0005	.0005	.0004	.0004	.0004	.0004	.0004	.0004	.0003
−3.2	.0007	.0007	.0006	.0006	.0006	.0006	.0006	.0005	.0005	.0005
−3.1	.0010	.0009	.0009	.0009	.0008	.0008	.0008	.0008	.0007	.0007
−3.0	.0013	.0013	.0013	.0012	.0012	.0011	.0011	.0011	.0010	.0010
−2.9	.0019	.0018	.0018	.0017	.0016	.0016	.0015	.0015	.0014	.0014
−2.8	.0026	.0025	.0024	.0023	.0023	.0022	.0021	.0021	.0020	.0019
−2.7	.0035	.0034	.0033	.0032	.0031	.0030	.0029	.0028	.0027	.0026
−2.6	.0047	.0045	.0044	.0043	.0041	.0040	.0039	.0038	.0037	.0036
−2.5	.0062	.0060	.0059	.0057	.0055	.0054	.0052	.0051	.0049	.0048
−2.4	.0082	.0080	.0078	.0075	.0073	.0071	.0069	.0068	.0066	.0064
−2.3	.0107	.0104	.0102	.0099	.0096	.0094	.0091	.0089	.0087	.0084
−2.2	.0139	.0136	.0132	.0129	.0125	.0122	.0119	.0116	.0113	.0110
−2.1	.0179	.0174	.0170	.0166	.0162	.0158	.0154	.0150	.0146	.0143
−2.0	.0228	.0222	.0217	.0212	.0207	.0202	.0197	.0192	.0188	.0183
−1.9	.0287	.0281	.0274	.0268	.0262	.0256	.0250	.0244	.0239	.0233
−1.8	.0359	.0351	.0344	.0336	.0329	.0322	.0314	.0307	.0301	.0294
−1.7	.0446	.0436	.0427	.0418	.0409	.0401	.0392	.0384	.0375	.0367
−1.6	.0548	.0537	.0526	.0516	.0505	.0495	.0485	.0475	.0465	.0455
−1.5	.0668	.0655	.0643	.0630	.0618	.0606	.0594	.0582	.0571	.0559
−1.4 3	.0808	.0793	.0778	.0764	.0749	.0735	.0721	.0708	.0694	.0681
−1.3	.0968	.0951	.0934	.0918	.0901	.0885	.0869	.0853	.0838	.0823
−1.2	.1151	.1131	.1112	.1093	.1075	.1056	.1038	.1020	.1003	.0985
−1.1	.1357	.1335	.1314	.1292	.1271	.1251	.1230	.1210	.1190	.1170
−1.0	.1587	.1562	.1539	.1515	.1492	.1469	.1446	.1423	.1401	.1379
−0.9	.1841	.1814	.1788	.1762	.1736	.1711	.1685	.1660	.1635	.1611
−0.8	.2119	.2090	.2061	.2033	.2005	.1977	.1949	.1922	.1894	.1867
−0.7	.2420	.2389	.2358	.2327	.2296	.2266	.2236	.2206	.2177	.2148
−0.6	.2743	.2709	.2676	.2643	.2611	.2578	.2546	.2514	.2483	.2451
−0.5	.3085	.3050	.3015	.2981	.2946	.2912	.2877	.2843	.2810	.2776
−0.4	.3446	.3409	.3372	.3336	.3300	.3264	.3228	.3192	.3156	.3121
−0.3	.3821	.3783	.3745	.3707	.3669	.3632	.3594	.3557	.3520	.3483
−0.2	.4207	.4168	.4129	.4090	.4052	.4013	.3974	.3936	.3897	.3859
−0.1	.4602	.4562	.4522	.4483	.4443	.4404	.4364	.4325	.4286	.4247
−0.0	.5000	.4960	.4920	.4880	.4840	.4801	.4761	.4721	.4681	.4641

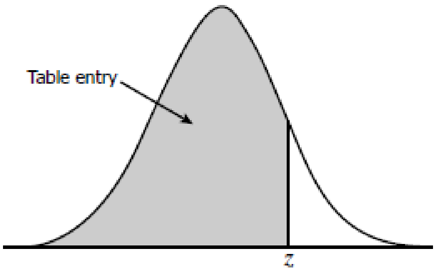


Table entry for z is the area under the standard normal curve to the left of z.

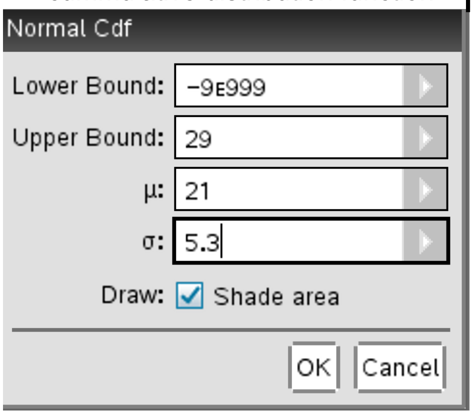
z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
0.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
0.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
0.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
0.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
0.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
0.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
0.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
0.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
0.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.14	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

calculator how to:

calculate it

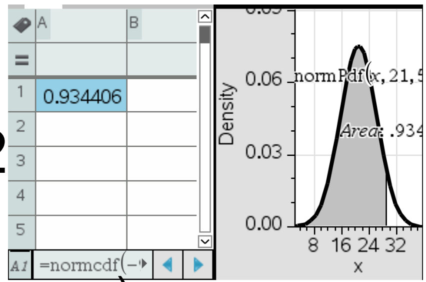
*in calc screen
menu
6-statistics
5- Distributions
2-normalcdf

"cummulative distribution function"

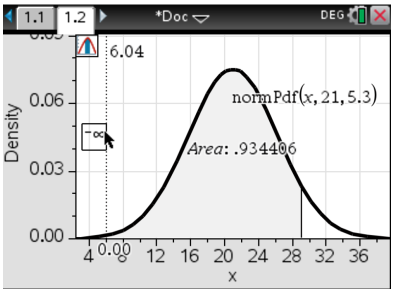


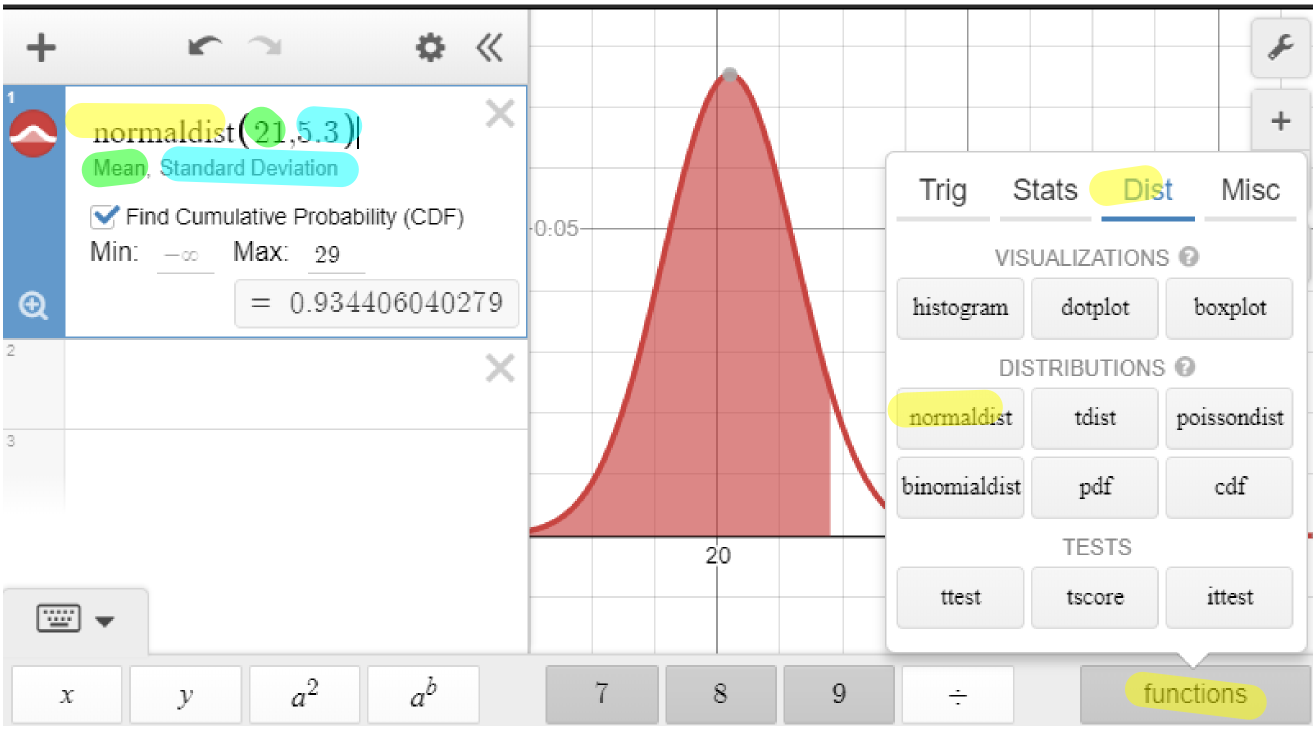
graph it

home, lists
menu-4-2-2



*ctrl,"up"(multi-screen)
right-click, ungroup, select graph.




normaldist(21,5.3)
Mean, Standard Deviation
Find Cumulative Probability (CDF)
Min: −∞ Max: 29
= 0.934406040279
0.05
20
Trig
Stats
Dist
Misc
VISUALIZATIONS
histogram
dotplot
boxplot
DISTRIBUTIONS
normaldist
tdist
poissondist
binomialdist
pdf
cdf
TESTS
ttest
tscore
ittest
functions

Date: ________________________ **Bell:** ______ **Homework 6:** Z-Scores & Standard Normal Distribution

**** This is a 2-page document! ****

1. The number of calories burned at the gym is normally distributed with a mean of 425 and a standard deviation of 51. Find the z-scores for each data value.

a) 268 **b)** 512 **c)** 450

2. The air pressure of each tire in a group is normally distributed with a mean of 39.2 pounds per square inch (psi) and a standard deviation of 2.5 psi. If a tire in this group has a z-score of -1.8, find its air pressure.

3. A set of math tests is normally distributed with a mean of 81 and a standard deviation of 5. If Adam's z-score was -0.6 and Leah's z-score was 2.2, how many points higher did Leah score than Adam?

4. The prices of various cereal boxes is normally distributed with a standard deviation of 0.5. If a box of Lucky Charms sells for $4.29 and has a z-score of 1.6, what is the mean price of a box of cereal?

5. The number of homes sold each year by a realtor is normally distributed with a mean of μ 54. If the realtor sold 36 (x) homes last year with a z-score of -2.4 (z), what is the standard deviation (σ)?

$$z = \frac{x - \mu}{\sigma}$$

$$-2.4 = \frac{(36 - 54)}{\sigma} = \frac{-18}{\sigma} = -2.4$$

$$\frac{-18}{-2.4} = \sigma$$

$$\sigma = 7.5$$

6. The last math exam of the year was normally distributed with a mean of 85 and a standard deviation of 5. Xavier needed at least a 95 on this test to get an A for the year. His teacher gave him his z-score: 1.8. Did Xavier get an A for the year?

7. The 400-meter race times recorded in the boys track meet was normally distributed with a standard deviation of 3 seconds. If David finished the race in 51.2 seconds with a z-score of -2.6, what was the mean time?

Use for questions 8-14: The weights of dogs in a dog show is normally distributed with a mean of 58 pounds and a standard deviation of 17.2 pounds. Use a standard normal distribution curve to find each probability.		
8. $P(z < -0.8)$	**9.** $P(z > 1.32)$	**10.** $P(-2.8 < z < -1.17)$

11. $P(X > 51)$	**12.** $P(35 < X < 62)$
13. $P(X < 102)$	**14.** $P(X > 85)$

15. The high temperatures in Virginia Beach during the month of July are normally distributed with a mean of 83° and a standard deviation of 6°. Find the probability that on a given day during the month, the temperature is greater than 92°.

16. The mean number of hours that the average person watches television each day is 4.18 hours with a standard deviation of 1.19 hours. Find the probability that someone watches between 3 and 5 hours per day.

$3 \le x \le 5$

$z = \frac{x-\mu}{\sigma}$

$z = \frac{3-4.18}{1.19}$ $z = \frac{5-4.18}{1.19}$

$z = -.99$ $z = .69$

16.11% 75.49%

$\begin{array}{r} 75.49 \\ -16.11 \\ \hline 59.38\% \end{array}$

between

17. The mean song length on Jack's iPad is 3 minutes and 3 seconds, with a standard deviation of 24 seconds. Find the probability that the next song to play is no more than 3 minutes and 45 seconds.