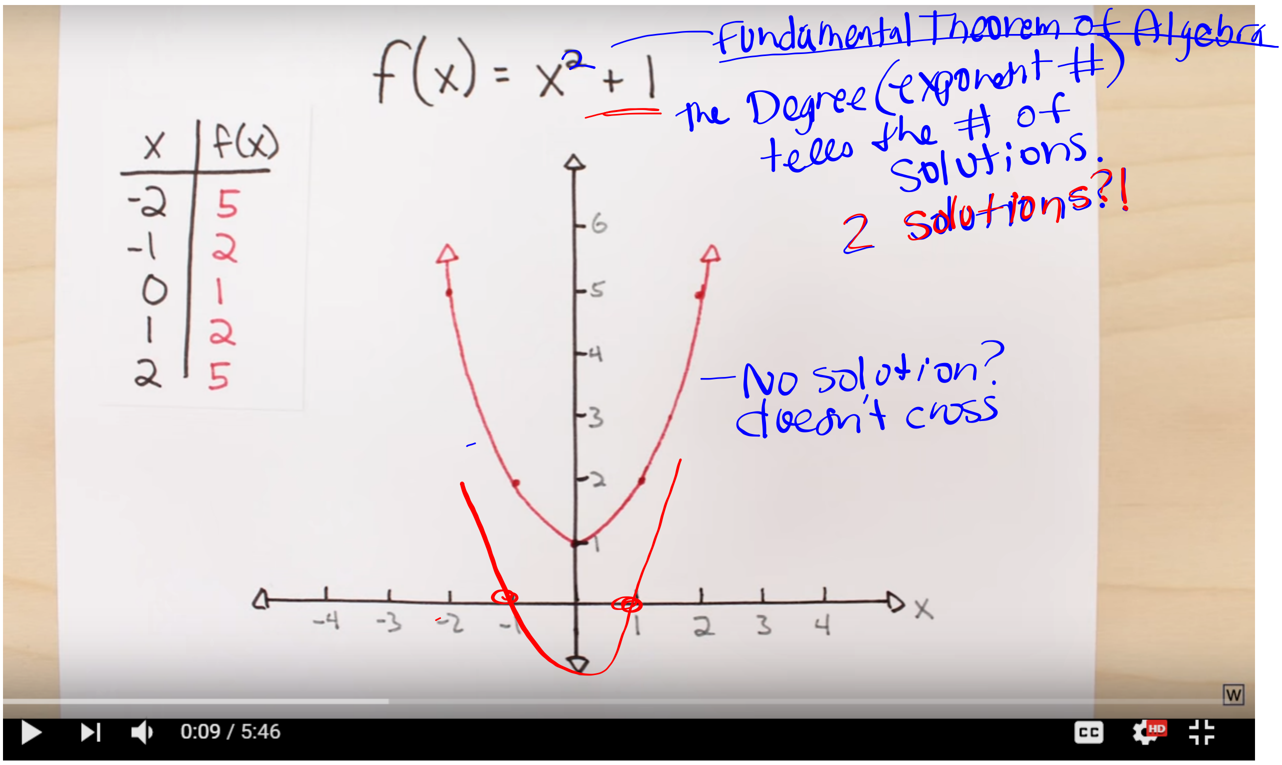
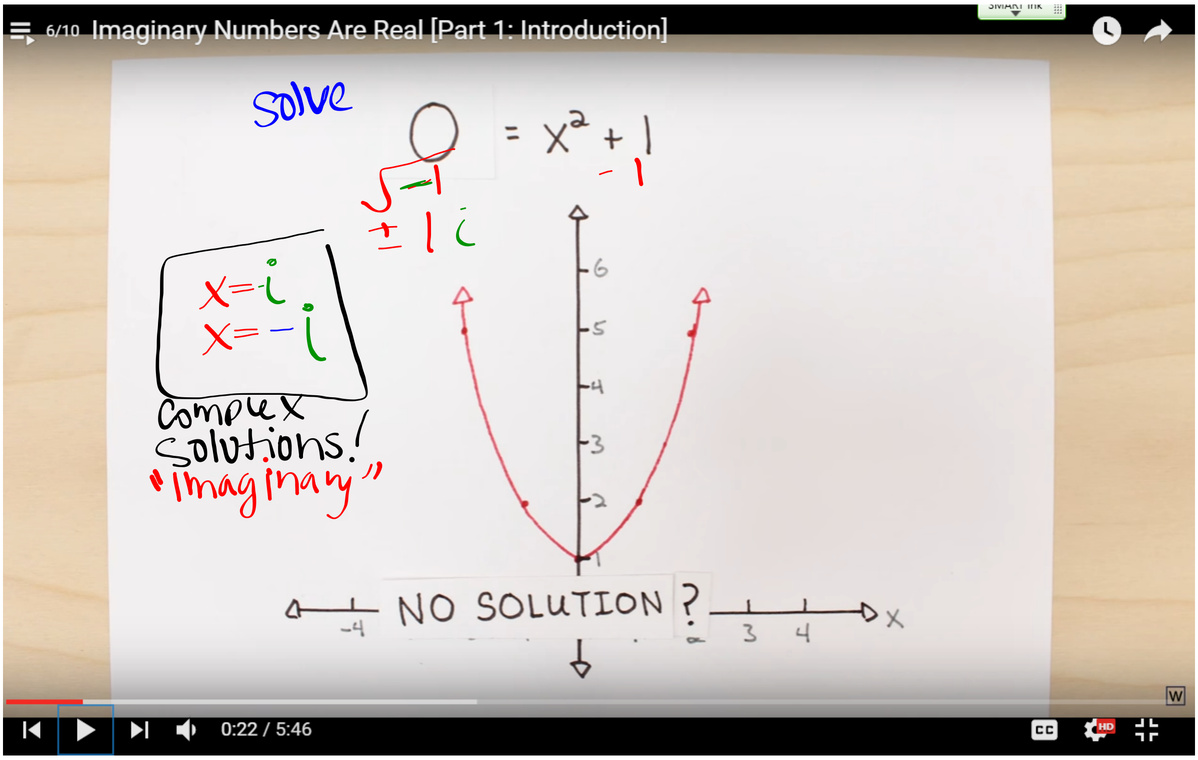
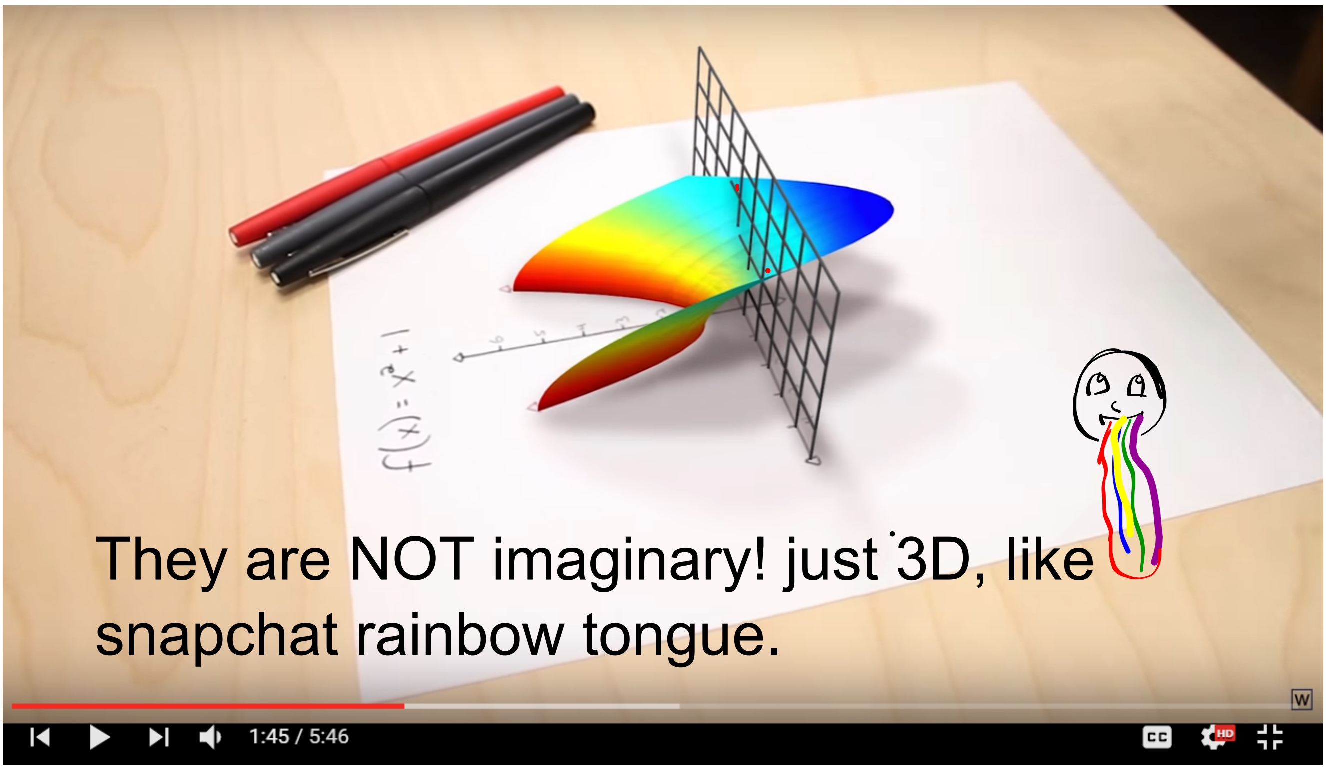

f(x) = x² + 1
Fundamental Theorem of Algebra
the Degree (exponent #) tells the # of solutions.
2 solutions?!
No solution? doesn't cross
x
f(x)
-2 5
-1 2
0 1
1 2
2 5
0:09 / 5:46


6/10 Imaginary Numbers Are Real [Part 1: Introduction]
Solve
0 = x² + 1
-1
√-1
±1i
x = i
x = -i
Complex solutions!
"Imaginary"
NO SOLUTION ?
x
0:22 / 5:46


They are NOT imaginary! just 3D, like snapchat rainbow tongue.
1:45 / 5:46

notes

$\cancel{\sqrt{-1} = i^2}$

$\boxed{-1 = i^2}$

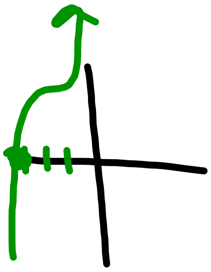
$x^2 + 1 = 0$
$-1 \quad -1$
$\sqrt{x^2} = \sqrt{-1}$
$x = \sqrt{-1}$
$= \sqrt{1}\, i$
$= \pm 1\, i$
$\boxed{= \pm i}$

Imaginary Numbers Practice Name:

$i^2 = -1. \quad i = \sqrt{-1}$

1. Practice multiplying this polynomial out into Standard form:

No touch!



$$f(x) = (x + 2i)(x - 2i)(x + 3) \quad -3$$

$x^2 \cancel{-2xi} \cancel{+2xi} - 4i^2 \quad (-1)$

$(x+3)(x^2+4)$

$\boxed{x^3 + 3x^2 + 4x + 12}$

2. What if you're given a complex number as a factor?

Example: $f(x) = x^3 + 7x^2 + 25x + 175$ and a factor is $(x - 5i)(x + 5i)$

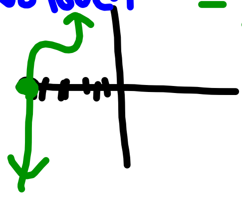
$x^2 + \cancel{5xi - 5xi} - 25i^2 \quad (-1)$

$(x^2 + 25)$

$$\begin{array}{r} x + 7 \\ x^2 + 25 \overline{)\, x^3 + 7x^2 + 25x + 175} \\ -x^3 \qquad\quad - 25x \qquad \\ \hline 7x^2 \quad 0x \quad 175 \\ -7x^2 \qquad\quad -175 \\ \hline 0 \end{array}$$

$(x-5i)(x+5i)(x+7)$

No touch $\quad -7$



**imaginary roots come in PAIRS

For problems 3-5, write down the possible combinations of real and complex roots the polynomial can have.

3. $f(x) = x^4 + 5x + 10$ 4-Real 4-real 4-real 2real+2i 4i
4. $f(x) = x^3 + 6$ 3-Real 1R:2i
5. $f(x) = 5x + 10$ 1Real

Find all of the factors.

6. $f(x) = x^3 - 6x^2 + 9x - 54$, a factor is $(x - 3i)(x+3i)$

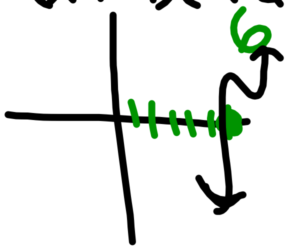
$-9i^2$

$-9(-1)$

x^2+9

$$
\begin{array}{r}
x^2 - 6 \\
x^2+9 \overline{\big)\, x^3 - 6x^2 + 9x - 54} \\
-(\cancel{x^3} \quad\quad\; + \cancel{9x}) \\
\hline
-6x^2 \quad 0x \;\; - 54 \\
-(-6x^2 \quad 0 \;\; - 54) \\
\hline
0
\end{array}
$$

$(x-3i)(x+3i)(x-6)$



Dividing Polynomials with Complex Roots

Instructions: For each problem, one factor of a cubic function is given. Find the remaining factors and use this information to determine all the roots of the function and sketch a graph.

1. $f(x) = x^3 + 3x^2 - 4x - 12$, $factor\ (x + 3)$

$x^2 - 4$

$x+3 \overline{)\, x^3 + 3x^2 - 4x - 12}$

$-(x^3 + 3x^2)$

$-4x - 12$

$-(-4x - 12)$

0

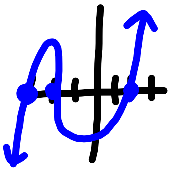
$x^2 - 4 = 0$

$\sqrt{x^2} = \sqrt{4}$

$x = \pm 2$

$(x+3)(x+2)(x-2)$

$-3 \quad -2 \quad 2$



2. $f(x) = x^3 + 6x^2 + 11x + 6$, $factor\ (x + 1)$

3. $f(x) = x^3 - 4x^2$ +9x-36 $factor\ (x - 3i)$

$(x+3i)$

$x^2 + 3ix - 3ix - 9i^2$

$(x^2 + 9)$

$x - 4$

$x^2+9 \overline{)\, x^3 - 4x^2 + 9x - 36}$

$-(x^3 + 9x)$

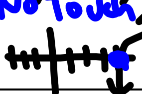
$-4x^2 \quad 0 \quad -36$

$-(-4x^2 - 36)$

0

$(x+3i)(x-3i)(x-4)$

No Touch +4



4. $f(x) = x^3 + 2x^2 + 5x + 10$, $factor\ (x + 2)$

$x^2 + 5$

$x+2 \overline{)\, x^3 + 2x^2 + 5x + 10}$

$-(x^3 + 2x^2)$

$5x + 10$

$-(5x + 10)$

0

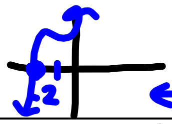
$x^2 + 5 = 0$

$\sqrt{x^2} = \sqrt{-5}$

$x = \pm\sqrt{5}i$

$(x+2)(x+\sqrt{5}i)(x-\sqrt{5}i)$

-2



5. $f(x) = 3x^3 + x^2$+108x-36 $factor\ (x + 6i)$

6. $f(x) = 2x^3 - 3x^2 + 4x - 6$, $factor\ (2x - 3)$

Instructions: Given the roots, find the factors and write the polynomial equation in standard form.

9. Roots: $3, -4, 0$

10. Roots: $5, 2i, -2i$

$(x-5)(x-2i)(x+2i)$

$(x-5)(x^2+4)$

$\boxed{x^3 - 5x^2 + 4x - 20}$

11. Roots: $\sqrt{3}, -\sqrt{3}, -2$

$(x-\sqrt{3})(x+\sqrt{3})(x+2)$

$x^2 + \sqrt{3}x - \sqrt{3}x - 3$

$\boxed{-\sqrt{3}\cdot\sqrt{3} = -(\sqrt{3})^2 = -3}$

$(x^2-3)(x+2)$

$\boxed{x^3 + 2x^2 - 3x - 6}$

12. Roots: $1, -1, -3$