7.2 Imagineering

A Develop Understanding Task

You are excited to get to vote on the plans for a proposed new thrill ride at a local theme park. The engineers want public input on the design for the new ride. You are one of ten teenagers who have been selected to review the plans based on your good math grades!

As your excitement mounts, the engineers begin their presentation. To your dismay, there are no models or illustrations of the proposed rides—each ride is described only with equations. The equations represent the path a rider would follow through the course of the ride.

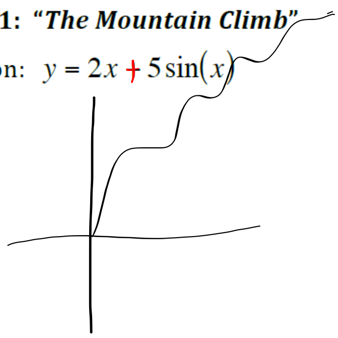
Unfortunately, your cell phone—which contains a graphing calculator app—is completely discharged due to too much texting and surfing the internet. So, you are trying hard to keep up with the presentation by trying to imagine what the graphs of each of these equations would look like. While each equation consists of functions you are familiar with, the combination of functions in each equation has you wondering about their combined effects.

For each of the following proposed thrill rides, use your imagination and best reasoning about the individual functions involved to sketch a graph of the path of the rider. Let *y* represent the height of the rider above the ground and *x* represent the distance from the start of the ride. Explain your reasoning about the shape of the graph. (Note: Use radians for trigonometric functions.)

Proposal #1: *"The Mountain Climb"*

The Equation: $y = 2x + 5\sin(x)$

My Graph:



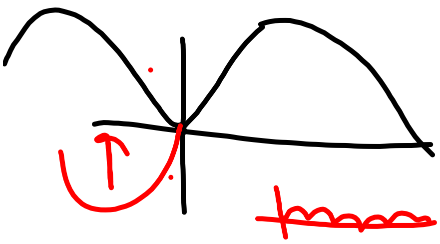
My Explanation:

sin graph that follow 2x

Proposal #2: *"The Periodic Bump"*

The Equation: $y = |10\sin(x)|$

My Graph:



My Explanation:

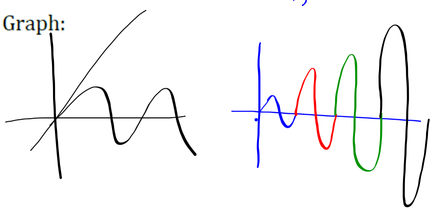
Sine function that can't go negative

Proposal #3: ***"The Amplifier"***

The Equation: $y = x \cdot \sin(x)$

1 sinx
2 sin
3

My Graph:



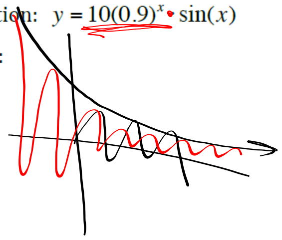
My Explanation:

increasing amplitude

Proposal #4: ***"The Gentle Wave"***

The Equation: $y = 10(0.9)^x \cdot \sin(x)$

My Graph:



My Explanation:

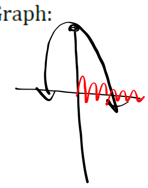
amp has an exponential decay.

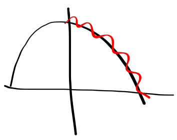
*fill

Proposal #5: ***"The Spinning High Dive"***

The Equation: $y = 100 - x^2 + 5\sin(4x)$

My Graph:





*follows (+)

My Explanation:

- parabola w/a sine wave

*follows

*fills (×)

When you got home your friends were all anxiously waiting to here about the proposed new rides. After explaining the situation, your friends all pull out their calculators and they began comparing your imagined images with the actual graphs.

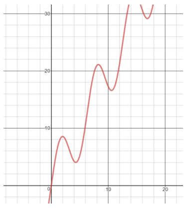
Some of your friends' graphs differed from the others because of their window settings. Some window settings revealed the features of the graphs you were expecting to see, while other window settings obscured those features.

Examine the actual graphs of each of the thrill ride proposals. Select a window setting that will reveal as many of the features of the graphs as possible. Explain any differences between your imagined graphs and the actual graphs. What features did you get right? What features did you miss?

Proposal #1: *"The Mountain Climb"*

The Equation: $y = 2x + 5\sin(x)$

Actual Graph:

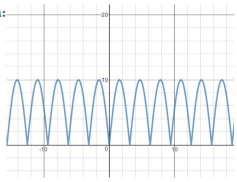


What features I got right and what I missed:

Proposal #2: *"The Periodic Bump"*

The Equation: $y = |10\sin(x)|$

Actual Graph:

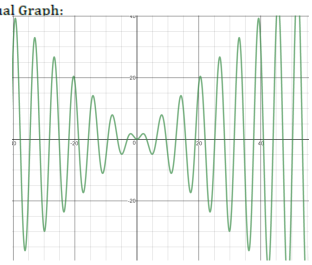


What features I got right and what I missed:

Proposal #3: *"The Amplifier"*

The Equation: $y = x \cdot \sin(x)$

Actual Graph:

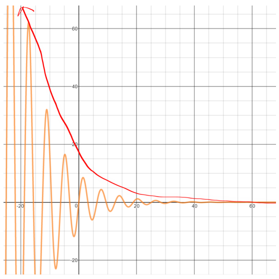


What features I got right and what I missed:

Proposal #4: *"The Gentle Wave"*

The Equation: $y = 10(0.9)^x \cdot \sin(x)$

Actual Graph:

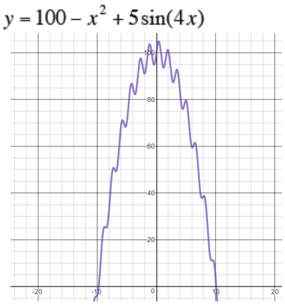


What features I got right and what I missed:

Proposal #5: *"The Spinning High Dive"*

The Equation: $y = 100 - x^2 + 5\sin(4x)$

Actual Graph:



What features I got right and what I missed:

You and your friends decide to propose a different ride to the engineers. Name your proposal and write its equation. Explain why you think the features of this graph would make a fun ride.

My Proposed Ride:

The equation for my ride:

My explanation of my proposal:

Name Modeling With Functions | 7.2

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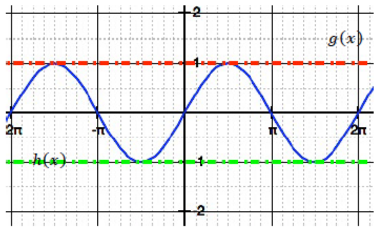
Ready, Set, Go!

Ready

Topic: Function boundaries.

1. The blue curve in the graph at the right shows the graph of $f(x) = sin(x)$.



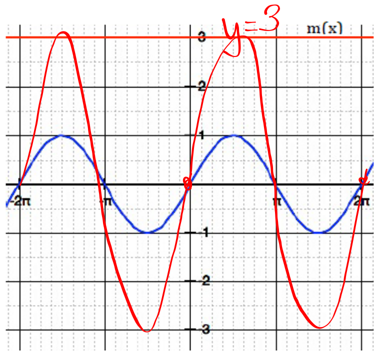
Write the equation of the dotted line labeled $g(x)$. (red) $y = 1$

Write the equation of the dotted line labeled $h(x)$. $y = -1$

List everything you notice about these three graphs. they all intersect

"Range" sinx has amp = 1 so the other lines are the boundaries

2. The blue curve in the graph at the right shows the graph of $f(x) = sin(x)$.



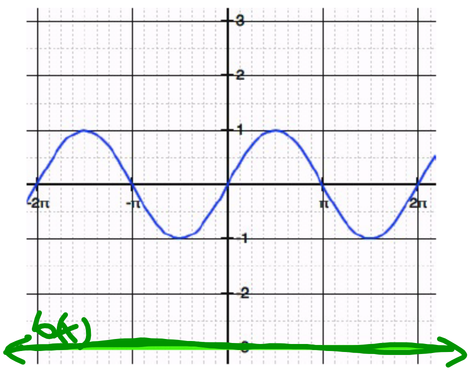
Write the equation of the line labeled $m(x)$. (red) $y = 3$

Sketch in the graph of $f(x) * m(x)$. $3\sin x$

What is the equation of $f(x) * m(x)$? $\sin x \cdot 3$

Would the line y = -3 also be a boundary line for your sketch? yes Explain. amp is (+) and (−)

3. The blue curve in the graph at the right shows the graph of $f(x) = sin(x)$.



Write the equation of the line labeled $b(x)$. (green)

Sketch in the graph of $f(x) * b(x)$.

What is the equation of $f(x) * b(x)$?

How does the graph of $f(x) * b(x)$ differ from the graph of $f(x) * m(x)$?

Would the line y = 3 also be a boundary line for your sketch? Explain.

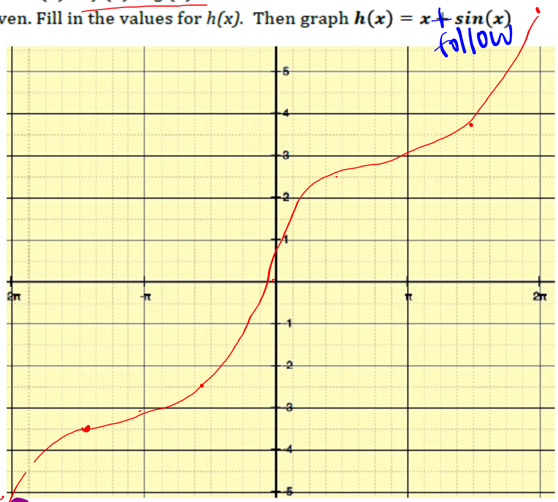
Set Topic: Combining functions

4. $f(x) = x$ $g(x) = \sin(x)$ $h(x) = f(x) + g(x)$

Some values for $f(x)$ and $g(x)$ are given. Fill in the values for $h(x)$. Then graph $\boldsymbol{h(x) = x + \sin(x)}$ with a smooth curve.

follow

x	$f(x)$	$g(x)$	$h(x)$
-2π	-6.28	0	-6.28
$-\frac{3\pi}{2}$	-4.71	1	-3.71
$-\pi$	-3.14	0	-3.14
$-\frac{\pi}{2}$	-1.57	-1	
0	0	0	
$\frac{\pi}{2}$	1.57	1	
π	3.14	0	
$\frac{3\pi}{2}$	4.71	-1	
2π	6.28	0	

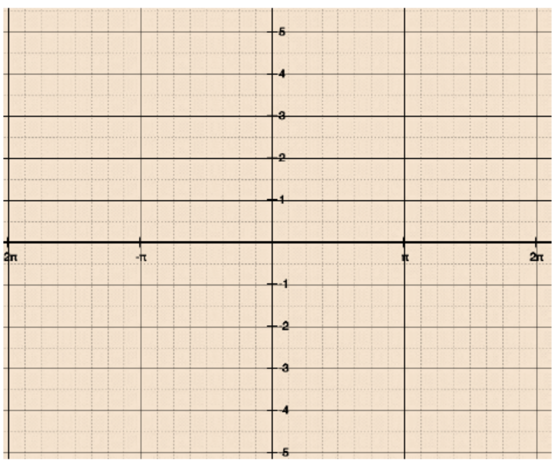


#5

5. $f(x) = x$ $g(x) = \sin(x)$ use f(x) & g(x) from table.

Now graph $k(x) = f(x) \cancel{\times} g(x)$ or $k(x) = x * \sin(x)$

x	$k(x)$
-2π	
$-\frac{3\pi}{2}$	
$-\pi$	
$-\frac{\pi}{2}$	
0	
$\frac{\pi}{2}$	
π	
$\frac{3\pi}{2}$	-4.71
2π	0



helped you match the equations.

6. $f(x) = |x^2 - 4|$
key features:

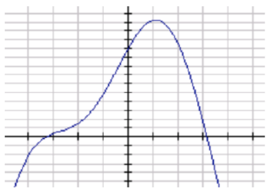
7. $g(x) = -x + 5\sin(x)$
key features:

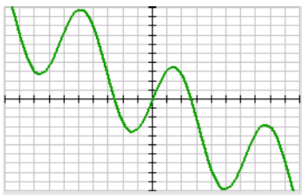
8. $h(x) = 4|\sin x|$
key features:

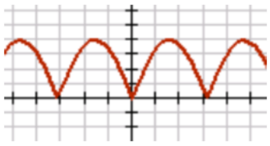
9. $d(x) = 10 - x^2 + 5\sin(x)$
key features:

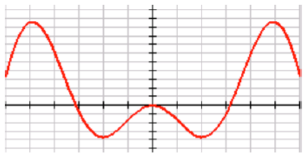
10. $w(x) = -x * 2\sin(x)$
key features:

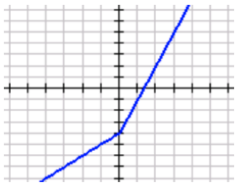
11. $r(x) = (2x - 4) + |x|$
key features:

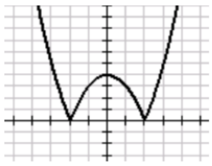
a)


b)


c)


d)


e)


f)


Go Topic: Families of functions

The chart below names five families of functions and the parent function. The parent is the equation in its simplest form. In the right hand column is a list of key features of the functions in random order. Match each key feature with the correct function. A key feature may relate to more than one function.

Family	*Parent(s)*	*Key features*
11. Linear	$y = x$	a) The ends of the graph have the same behavior. b) The graphs have a horizontal asymptote and a vertical asymptote.
12. Quadratic a,	$y = x^2$	c) The graph only has a horizontal asymptote. d) These functions either have both a local maximum and minimum or no local maximum and minimum.
13. Cubic	$y = x^3$	e) The graph is usually defined in terms of its slope and y-intercept. f) The graph has either a maximum or a minimum but not both.
14. Exponential g.	$y = 2^x$ $y = 3^x$ etc.	g) As x approaches -∞, the function values approach the x-axis. $-\infty$ h) The ends of the graph have opposite behavior. i) The rate of change of this graph is constant. j) The rate of change of this graph is constantly changing.
15. Rational b, g.	$y = \frac{1}{x}$ etc.	k) This graph has a linear rate of change. l) These functions are of degree 3. m) The variable is an exponent. n) These functions contain fractions with a polynomial in both the numerator and denominator. p) The constant will always be the y-intercept.

constant
$x + 2$
$x^2 + 2$
$x^3 + 2 \ldots$