1.4 Pulling a Rabbit Out of a Hat

A Solidify Understanding Task

I have a magic trick for you:

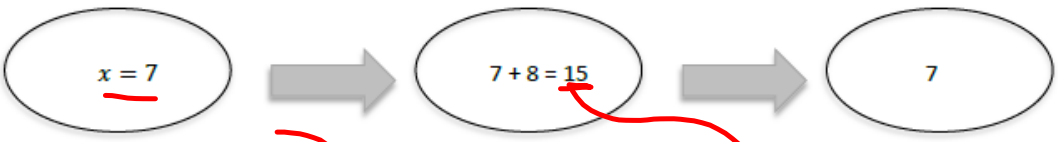
- Pick a number, any number.
- Add 6
- Multiply by the result by 2
- Subtract 12
- Divide by 2
- The answer is the number you started with!

People are often mystified by such tricks but those of us who have studied inverse operations and inverse functions can easily figure out how they work and even create our own number tricks. Let's get started by figuring out how inverse functions work together.

For each of the following function machines, decide what function can be used to make the output the same as the input number. Describe the operation in words and then write it symbolically.

Here's an example:

Input Output



$f(x) = x + 8$

$f^{-1}(x) = x - 8$

In words: Subtract 8 from the result

1.

Input → Output

$x = 7$ → $3 \cdot 7 = 21$ → 7

$f^{-1}(x) = \frac{x}{3}$

check $\frac{21}{3} = 7$ ✓

2.

$f(x) = 3x$

$\frac{y}{3} = \frac{3x}{3}$

$\frac{y}{3} = x$ $\Leftrightarrow y = \frac{x}{3}$

In words: divide by 3

Input → Output

$x = 7$ → $7^2 = 49$ → 7

$f(x) = x^2$

$\sqrt{y} = x$

$\sqrt{y} = x \Leftrightarrow y = \sqrt{x}$

$f^{-1}(x) = \sqrt{x}$

check $\sqrt{49} = 7$ ✓

In words: square rooting

$x^8 = \sqrt[8]{x}$

$x^2 \Leftrightarrow \sqrt{x}$

$2^x \Leftrightarrow \log$

Input → Output

3.

$x = 7$ → $2^7 = 128$ → 7

$\log_2 128 = 7$

$f(x) = 2^x$

$\log_2 y = x$

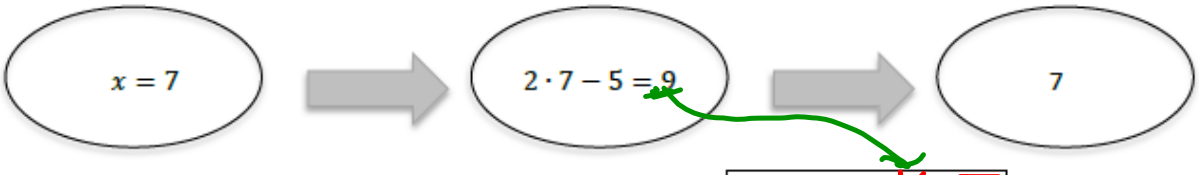
$\log_2 y = x \Leftrightarrow y = \log_2 x$

$f^{-1}(x) = \log_2 x$

In words:

4.

Input Output



$f(x) = 2x - 5$

$y = 2x - 5$

$+5 \quad +5$

$y + 5 = 2x$

$\frac{y+5}{2} = x$

$f^{-1}(x) = \frac{x+5}{2}$

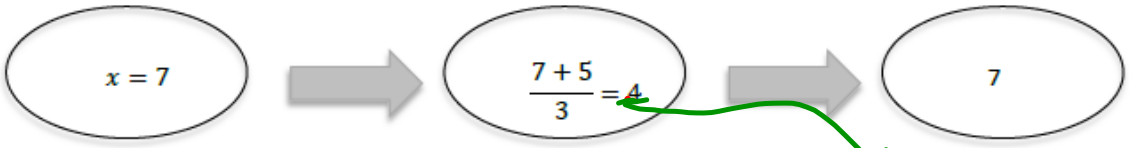
In words: add 5 divide by 2

check $\frac{9+5}{2} = 7$ ✓

$\Leftrightarrow y = \frac{x+5}{2}$

5.

Input Output



PEMDAS

$f(x) = \frac{(x+5)}{3}$

$3 \cdot y = \frac{(x+5)}{3} \cdot 3$

$3y = x + 5$

$-5 \quad -5$

$3y - 5 = x$

$f^{-1}(x) = 3x - 5$

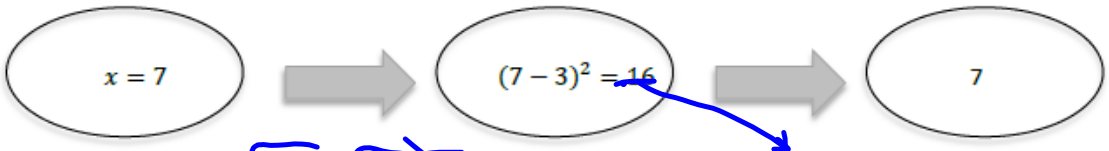
$3(4) - 5 = 7$ ✓

In words: Multi 3, sub 5

$\Leftrightarrow y = 3x - 5$

6.

Input Output



$f(x) = \sqrt{(x-3)^2}$

$\sqrt{y} = x - 3$

$+3 \quad +3$

$\sqrt{y} + 3 = x$

$f^{-1}(x) = \sqrt{x} + 3$

$\sqrt{16} + 3$

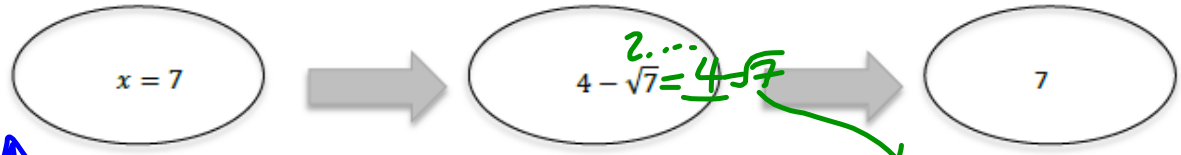
$4 + 3 = 7$ ✓

In words: square root & add 3

$\Leftrightarrow y = \sqrt{x} + 3$

7.

Input Output



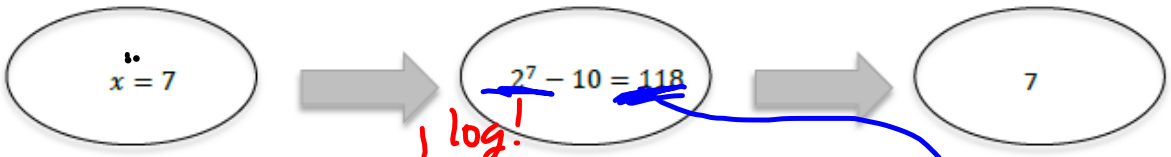
$f(x) = 4 - \sqrt{x}$

$f^{-1}(x) = (-x+4)^2$

In words:

8.

Input Output



$f(x) = 2^x - 10$

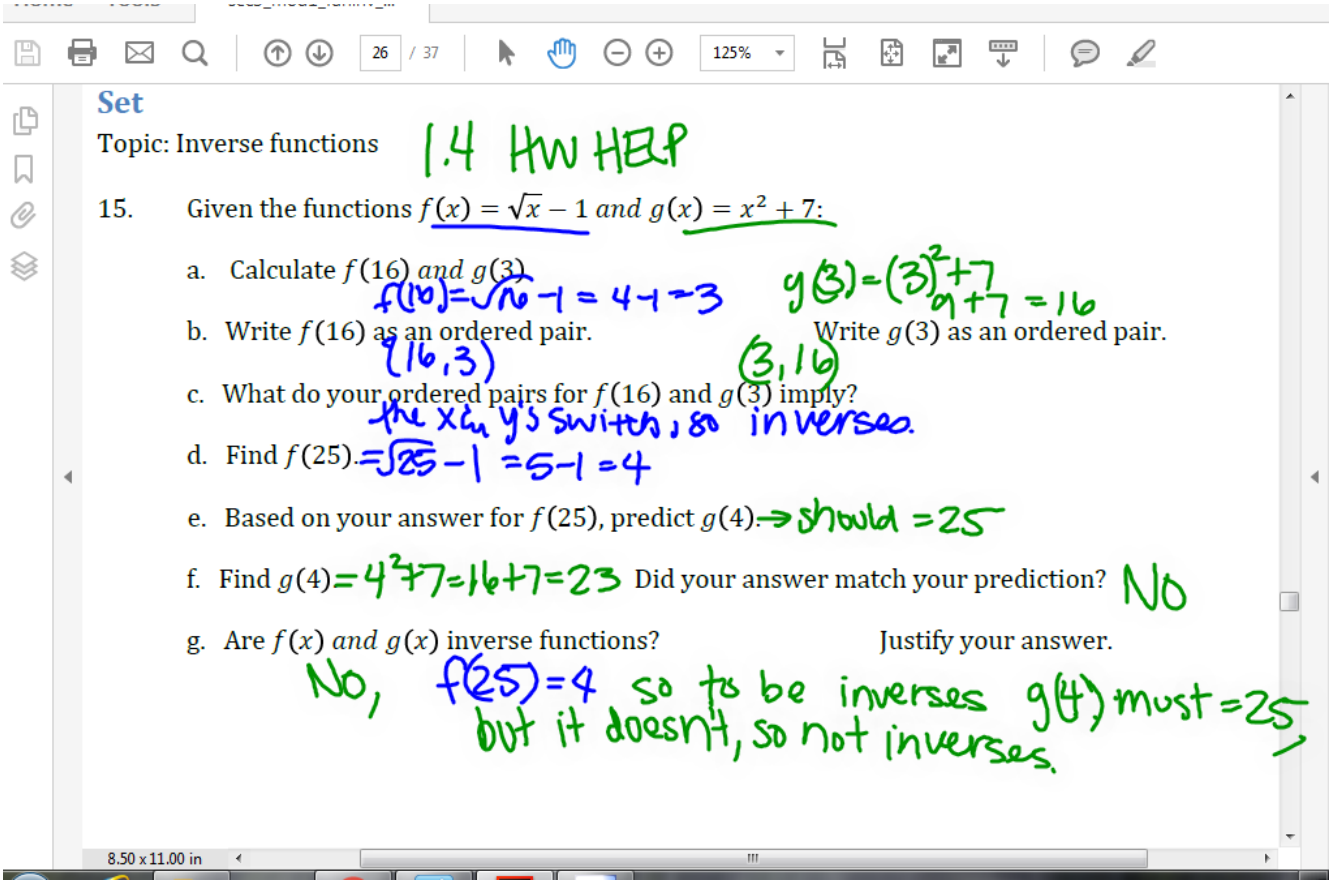
$f^{-1}(x) = \log_2(x+10)$

In words:

9. Each of these problems began with $x = 7$. What is the difference between the x used in $f(x)$ and the x used in $f^{-1}(x)$?

10. In #6, could any value of x be used in $f(x)$ and still give the same output from $f^{-1}(x)$? Explain. What about #7?

11. Based on your work in this task and the other tasks in this module what relationships do you see between functions and their inverses?



Set

Topic: Inverse functions 1.4 HW HELP

15. Given the functions $f(x) = \sqrt{x} - 1$ *and* $g(x) = x^2 + 7$:

a. Calculate $f(16)$ *and* $g(3)$. $f(16)=\sqrt{16}-1=4-1=3$ $g(3)=(3)^2+7$ $9+7=16$

b. Write $f(16)$ as an ordered pair. (16,3) Write $g(3)$ as an ordered pair. (3,16)

c. What do your ordered pairs for $f(16)$ and $g(3)$ imply? the x&y's switch, so inverses.

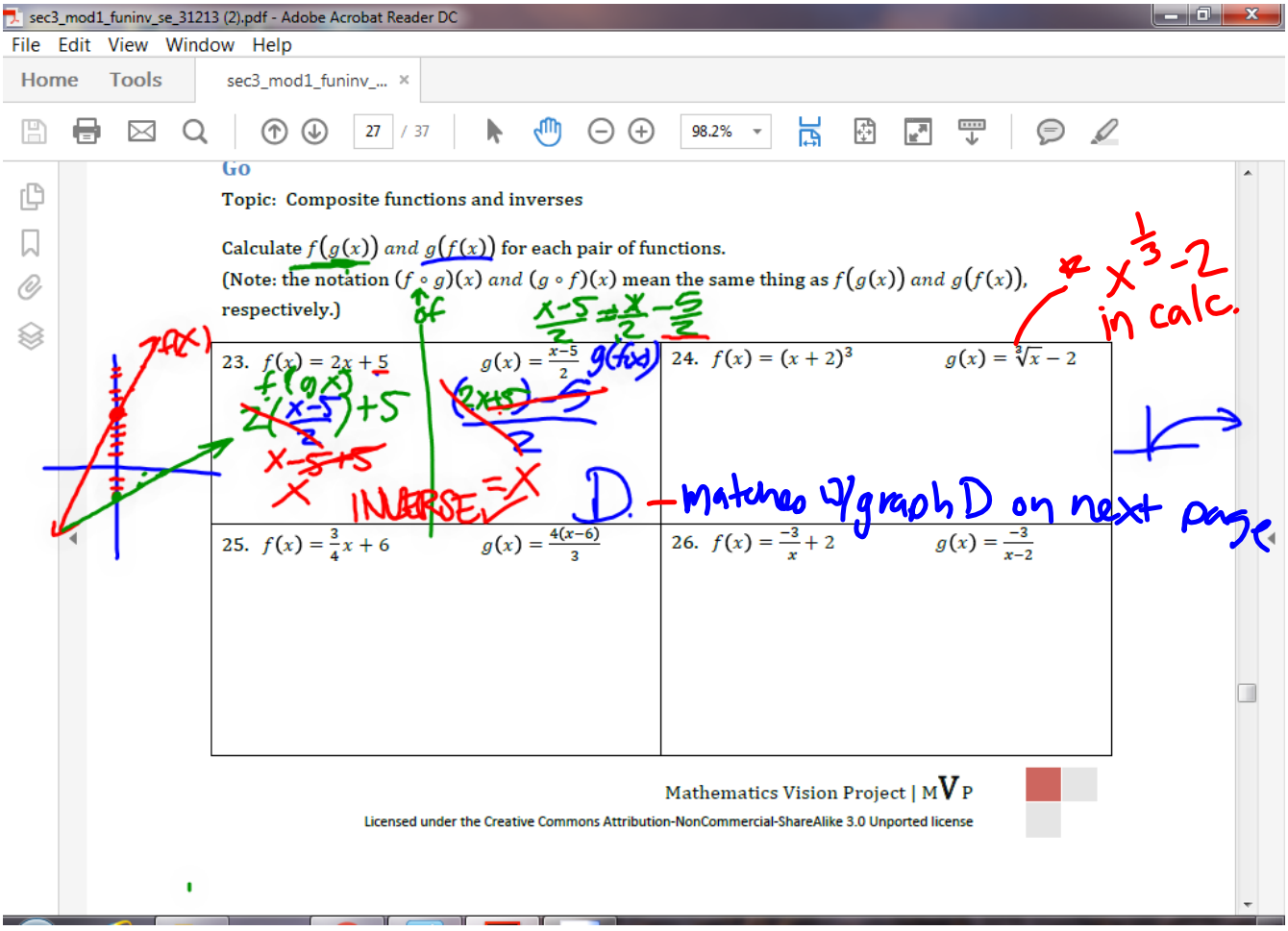
d. Find $f(25)$. $=\sqrt{25}-1=5-1=4$

e. Based on your answer for $f(25)$, predict $g(4)$. → should =25

f. Find $g(4)=4^2+7=16+7=23$ Did your answer match your prediction? No

g. Are $f(x)$ *and* $g(x)$ inverse functions? Justify your answer.

No, $f(25)=4$ so to be inverses $g(4)$ must $=25$, but it doesn't, so not inverses.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Go
Topic: Composite functions and inverses
Calculate $f(g(x))$ and $g(f(x))$ for each pair of functions.
(Note: the notation $(f \circ g)(x)$ and $(g \circ f)(x)$ mean the same thing as $f(g(x))$ and $g(f(x))$, respectively.)
23. $f(x) = 2x + 5$ $g(x) = \frac{x-5}{2}$
24. $f(x) = (x+2)^3$ $g(x) = \sqrt[3]{x} - 2$
25. $f(x) = \frac{3}{4}x + 6$ $g(x) = \frac{4(x-6)}{3}$
26. $f(x) = \frac{-3}{x} + 2$ $g(x) = \frac{-3}{x-2}$
INVERSE
D. - matches W/graph D on next page
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