1.3 Tracking the Tortoise

A Solidify Understanding Task

You may remember a task from last year about the famous race between the tortoise and the hare. In the children's story of the tortoise and the hare, the hare mocks the tortoise for being slow. The tortoise replies, "Slow and steady wins the race." The hare says, "We'll just see about that," and challenges the tortoise to a race.

In the task, we modeled the distance from the starting line that both the tortoise and the hare travelled during the race. Today we will consider only the journey of the tortoise in the race.

Because the hare is so confident that he can beat the tortoise, he gives the tortoise a 1 meter head start. The distance from the starting line of the tortoise including the head start is given by the function:

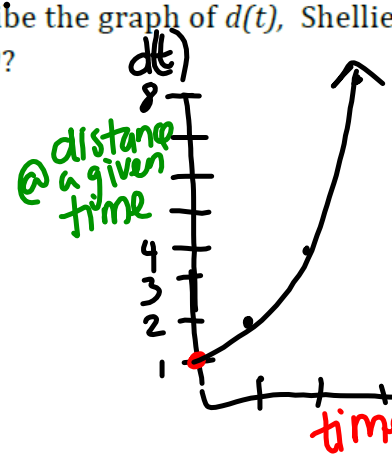
$d(t) = 2^t$ (d in meters and t in seconds)

The tortoise family decides to watch the race from the sidelines so that they can see their darling tortoise sister, Shellie, prove the value of persistence.

1. How far away from the starting line must the family be, to be located in the right place for Shellie to run by 10 seconds after the beginning of the race? After 20 seconds?

$d(t)$ $d=?$ 2^{10} given $d(10) = 2^{10}$ $t=10$ $= 2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2\cdot2$ $= 1024$ meters $t=20$ $2^{20} = 1048576$

2. Describe the graph of $d(t)$, Shellie's distance at time t. What are the important features of $d(t)$?



t	$d(t)$
0	$2^0 = 1$
1	$2^1 = 2$
2	$2^2 = 4$
3	$2^3 = 8$

)x2

- very steep
- begins at (0,1)
- seems exponential
- never negative
- doubles, x2
- distance depends on the time

inverse!

3. If the tortoise family plans to watch the race at 64 meters away from Shellie's starting point, how long will they have to wait to see Shellie run past?

d=

t=?

$64 = 2^t$

$2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 2^6$

2, 4, 8, 16, 32, 64

time 6 seconds

t=?

4. How long must they wait to see Shellie run by if they stand 1024 meters away from her starting point?

d=1024

$2^t = 1024$

$2^{10} = 1024$

$t = 10$

last page

output y

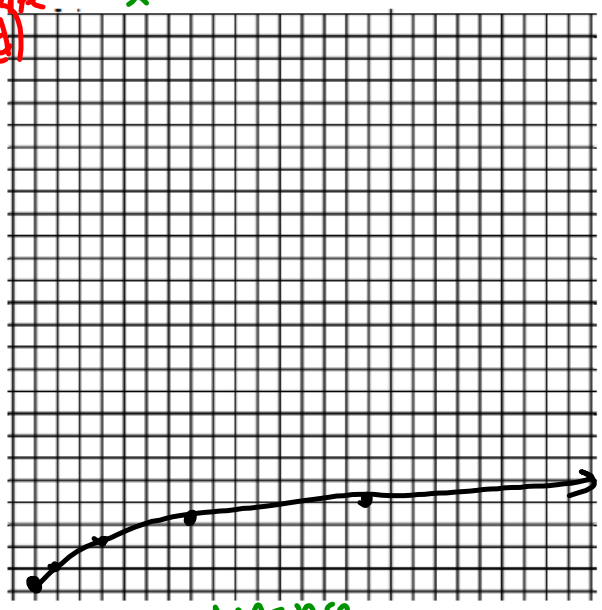
5. Draw a graph that shows how long the tortoise family will wait to see Shellie run by at a given location from her starting point.

time given distance

d	t(d)
1	0
2	1
4	2
8	3
16	4

time t(d)

x



distance

t=?

6. How long must the family wait to see Shellie run by if they stand 220 meters away from her starting point?

Estimate!

$2^t = 220$

2^6, 2^7, 2^8, ...

64, 128, 256

in between 7-8 seconds

7. What is the relationship between $d(t)$ and the graph that you have just drawn? How did you use $d(t)$ to draw the graph in #5?

A. B. B. switched x & y

d(t) is the inverse of t(d)

$d^{-1}(t) = t(d)$

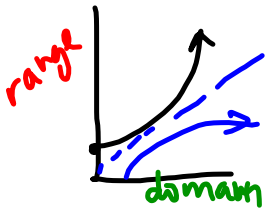
time

time

distance

8. Consider the function $f(x) = 2^x$.

A) What are the domain and range of $f(x)$? B. Is $f(x)$ invertible?



domain: $\mathbb{R}, x \geq 0$

range: $\mathbb{R}, y \geq 1$

yes invertible

B) Graph $f(x)$ and $f^{-1}(x)$ on the grid below.

range

domain

This is a Log function! $y = \log_a x$

C) What are the domain and range of $f^{-1}(x)$?

D: $\mathbb{R}, x \geq 1$

R: $\mathbb{R}: y \geq 0$

switched!

9. If $f(3) = 8$, what is $f^{-1}(8)$? How do you know?

$f(3) \Rightarrow 8$

$3 \Leftarrow f^{-1}(8)$

Because x & y switch if inverses.

10. If $f\left(\frac{1}{2}\right) = 1.414$, what is $f^{-1}(1.414)$? How do you know?

$f\left(\frac{1}{2}\right) \rightarrow 1.414$

$\frac{1}{2} \leftarrow f^{-1}(1.414$

Because x & y switch if inverses.

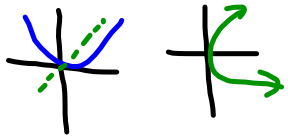
11. If $f(a) = b$ what is $f^{-1}(b)$? Will your answer change if $f(x)$ is a different function? Explain.

A. $f(a) \rightarrow b$

$a \leftarrow f^{-1}(b)$

B. Yes, can change because some inverses are not functions. They don't pass the vertical line test.

Ex:



$\star \quad f(f^{-1}(x)) = x$

Name Functions and their Inverses | 1

Homework Help 1.3

Ready, Set, Go!

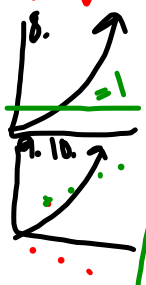
Ready

Topic: Solving Exponential Equations

Solve for the value of x.

1. $5^{x+1} = 5^{2x-3}$ $x+1=2x-3$, $+3 \quad +3$, $4=x$
2. $7^{3x-2} = 7^{-2x+8}$
3. $4^{3x} = 2^{2x-8}$ $3^{2(2x-3)}$
4. $3^{5x-4} = 9^{2x-3}$
5. $8^{x+1} = 2^{2x+3}$ $2^{3(x+1)} = 2^{2x+3}$, $3x+3 = 2x+3$, $-2x-3 \quad -2x-3$, $x=0$
6. $5^{x} = \frac{1}{125}$
7. $3^{x+1} = \frac{1}{81}$ 3^{-4}, $x+1=-4$, $-1 \quad -1$, $x=-5$

8-10 IN CLASS SKIP



Topic: Writing the logarithmic form of an exponential equation.

Definition of Logarithm: *For all positive numbers a, where a ≠ 1, and all positive numbers x,*

$y = \log_a x$ *means the same as* $x = a^y$.

(Note the ***base*** *of the exponent and the* ***base*** *of the logarithm are both* ***a****.)*

8. Why is it important that the definition of logarithms states that the base of the logarithm does not equal 1?

What if a=1, why bad? $1^y = x$

y	1	2	3	4
x	1	1	1	1

a ≠ 1? a always equals 1, so not exponential, bad. If it is exponential it is alternating

9. Why is it important that the definition states that the base of the logarithm is positive?

a > 0, positive, what if a is (−)? why bad? $(-3)^{1/3}, (-3)^{2/3}, (-3)^{3/3}, (-3)^{4/3}$

1	2	3	4
−	+	−	+

Not exponential

10. Why is it necessary that the definition states that x in the expression $\log_a x$ is positive?

The x in $\log_a x$ must be positive, because it is the y (answers) to the exponential, if not we get this.

Write the following exponential equations in logarithmic form.

Exponential form	Logarithmic form
11. $5^4 = 625$	$4 = \log_5 625$
12. $3^2 = 9$	
13. $\left(\frac{1}{2}\right)^{-3} = 8$	
14. $10^4 = 10000$	
15. $4^{-2} = \frac{1}{16}$	
16. $e^7 = e^7$	$7 = \log_e e^x$
~~17. $a^y = x \log_a x = y$~~ typo ignore.	

$a^y = x$ $y = \log_a x$

18. Compare the exponential form of an equation to the logarithmic form of an equation. What part of the exponential equation is the **answer** to the logarithmic equation?

the answer is the exponent.

TYPO fix

Go

Topic: Evaluating functions.

The functions $f(x), g(x), and\ h(x)$ are defined below.

$f(x) = -2x$ $g(x) = 2x + 5$ $h(x) = x^2 + 3x - 10$

Calculate the indicated function values. Simplify your answers.

19. f(a) 20. $f(b^2)$ 21. f(a + b) 22. $f(g(x))$

23. g(a) 24. $g(b^2)$ 25. g(a + b) 26. $h(f(x))$

27. h(a) 28. $h(b^2)$ 29. h(a + b) 30. $h(g(x))$

use g(x) replace x with b^2

$g(b^2) = 2(b^2) + 5$

$= 2b^2 + 5$

use h replace x's with f(x) equation, so $-2x$.

$h(-2x) = (-2x)^2 + 3(-2x) - 10$

$= 4x^2 - 6x - 10$

$h(2x+5) = (2x+5)^2 + 3(2x+5) - 10$

$(2x+5)(2x+5)$

$4x^2 + 10x + 10x + 25$

$20x$

$\rightarrow = 4x^2 + 20x + 25 + 6x + 15 - 10$

$= 4x^2 + 26x + 30$