1.2 Flipping Ferraris

A Solidify Understanding Task

When people first learn to drive, they are often told that the faster they are driving, the longer it will take to stop. So, when you're driving on the freeway, you should leave more space between your car and the car in front of you than when you are driving slowly through a neighborhood. Have you ever wondered about the relationship between how fast you are driving and how far you travel before you stop, after hitting the brakes?

1. Think about it for a minute. What factors do you think might make a difference in how far a car travels after hitting the brakes? good tires, brakes, weight

weather; snow, ice, rain uphill, road type, etc.

There has actually been quite a bit of experimental work done (mostly by police departments and insurance companies) to be able to mathematically model the relationship between the speed of a car and the braking distance (how far the car goes until it stops after the driver hits the brakes).

2. Imagine your dream car. Maybe it is a Ferrari 550 Maranello, a super-fast Italian car. Experiments have shown that on smooth, dry roads, the relationship between the braking distance (d) and speed (s) is given by $\boxed{d(s) = 0.03s^2}$. Speed is given in miles/hour and the distance is in feet. Calc. distance given speed.

   a) How many feet should you leave between you and the car in front of you if you are driving the Ferrari at 55 mi/hr? = s

d=?

calculate distance given speed is 55 $d(s) = .03s^2$

$d(55) = .03(55)^2$

So if you are going 55mph, you need ~91 feet to stop. → $d = 90.75$ ft d=?

b) What distance should you keep between you and the car in front of you if you are driving at 100 mi/hr? =S

$d(s) = .03s^2$

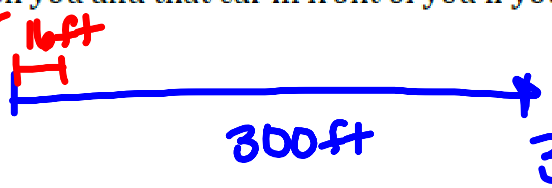
$d(100) = .03(100)^2$

$= .03 \times 10000$

$= 300$ ft

Not part of calculation*
-means: distance required given speed of 100

c) If an average car is about 16 feet long, about how many car lengths should you have between you and that car in front of you if you are driving 100 mi/hr?



needs distance of 300ft.

$300 \div 16 = 18.75$

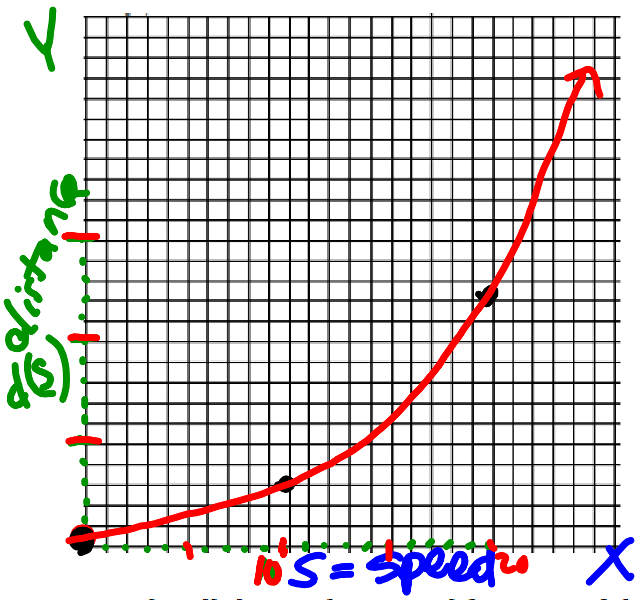
$= 19$ whole cars

d) It makes sense to a lot of people that if the car is moving at some speed and then goes twice as fast, the braking distance will be twice as far. Is that true? Explain why or why not.

Well looking back in and a) 55mph needs 91ft
b) 100mph needs 300ft

speed is almost twice
but distance is NOT twice, its more like 3 times.
SO NOT True.

3. Graph the relationship between braking distance $d(s)$, and speed (s), below.



given x
depends on speed
distance given speed Y

make a table to help graph

s	d(s)
0	0
10	3
20	12
30	27 ← $.03(30)^2$
40	48
50	75

4. Describe all the mathematical features of the relationship between braking distance and speed for the Ferrari modeled by $d(s) = 0.03s^2$.

goes through (0,0) the origin
domain: $x \geq 0$
range: $y \geq 0$
+ increasing non linear → exponential

5. What if the driver of the Ferrari 550 was cruising along and suddenly hit the brakes to stop because she saw a cat in the road? She skidded to a stop, and fortunately, missed the cat. When she got out of the car she measured the skid marks left by the car so that she knew that her braking distance was 31 ft. = d

s = ?

a) How fast was she going when she hit the brakes?

$$\sqrt{\frac{31}{.03}} = \frac{\cancel{.03}s^2}{\cancel{.03}}$$

$$\sqrt{\frac{31}{.03}} = 32.15 \text{ mph} \leftarrow \sqrt{\frac{31}{.03}} = s$$

c) If she didn't see the cat until she was 15 feet away, what is the fastest speed she could be traveling before she hit the brakes if she wants to avoid hitting the cat?

$d = .03s^2$ $\sqrt{\frac{15}{.03}} = 22.36\,m/h$

6. Part of the job of police officers is to investigate traffic accidents to determine what caused the accident and which driver was at fault. They measure the braking distance using skid marks and calculate speeds using the mathematical relationships just like we have here, although they often use different formulas to account for various factors such as road conditions. Let's go back to the Ferrari on a smooth, dry road since we know the relationship. Create a table that shows the speed the car was traveling based upon the braking distance.

speed given distance

d	s(d)
0	0
3	10
12	20
27	30
48	40

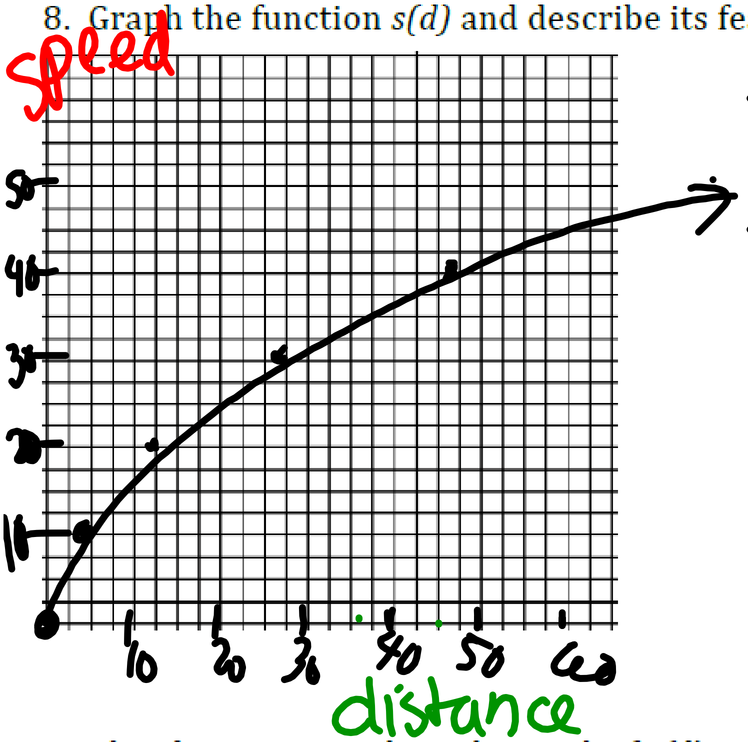
7. Write an equation of the function *s(d)* that gives the speed the car was traveling for a given braking distance.

$$\frac{d}{.03} = \frac{.03s^2}{\cancel{.03}}$$

$$\sqrt{\frac{d}{.03}} = \sqrt{s^{\cancel{2}}}$$

$$\sqrt{\frac{d}{.03}} = s(d)$$

8. Graph the function $s(d)$ and describe its features.



- starts at (0,0)
- increasing
- flattens out
- domain ℝ $x \geq 0$
- range ℝ $y \geq 0$

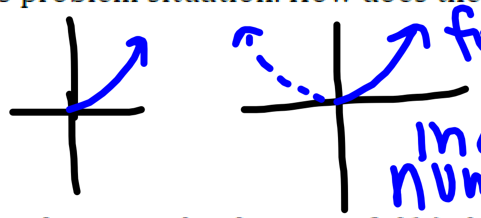
Mathematics Vision Project | MVP

9. What do you notice about the graph of $s(d)$ compared to the graph of $d(s)$? What is the relationship between the functions $d(s)$ and $s(d)$?

They are opposite x's & y's. reflected over y=x. INVERSES!

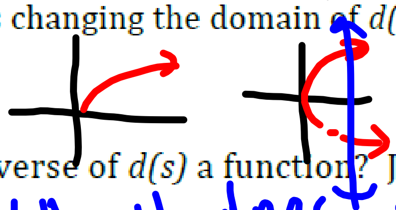
10. Consider the function $d(s) = 0.03s^2$ over the domain of all real numbers, not just the domain of this problem situation. How does the graph change from the graph of $d(s)$ in question #3?

d(s)



for ℝ include negative numbers

11. How does changing the domain of $d(s)$ change the graph of the inverse of $d(s)$?



12. Is the inverse of $d(s)$ a function? Justify your answer.

NO, it does NOT pass verticle line test.

Exponent Review

$2^{-3} = \frac{1}{2\cdot 2\cdot 2} = \frac{1}{8}$

$2^{-2} = \frac{1}{2\cdot 2} = \frac{1}{4}$

$2^{-1} = \frac{1}{2} = \frac{1}{2}$

$2^{0} = \frac{2}{2} = 1$

$2^{1} = 2 = 2$

$2^{2} = 2\cdot 2 = 4$

$2^{3} = 2\cdot 2\cdot 2 = 8$

$\div 2$

$\times 2$

Name Functions and their Inverses | 1.2

Homework Help 1.2

Ready, Set, Go!

Ready

Topic: Solving for a variable

Solve for x.

1. $17 = 5x + 2$

easier factoring

2. $2x^2 - 5 = 3x^2 - 12x + 31$

$-2x^2 \quad +5 \quad -2x^2 \quad +5$

want $ax^2+bx+c=0$

$0 = 1x^2 - 12x + 36$

$(x-6)(x-6)$

$1 \cdot 36$, 36, -6, -6, -12

$\boxed{x = 6, 6} \rightarrow \boxed{x = 6}$

3. $11 = \sqrt{2x+1}$

harder

4. $\sqrt{2x^2+x-2} = 2$

want: $Ax^2+bx+c=0$

$2x^2 + x - 2 = 4$

$-4 \quad -4$

$2x^2 + x - 6 = 0$

$2 \cdot -6$, -12, 4, -3, 1

$2x^2 \; 4x - 3x - 6 = 0$

$2x(x+2) - 3(x+2) \rightarrow (x+2)(2x-3) = 0$

$\boxed{x = -2, \; 3/2}$

5. $-4 = \sqrt[3]{5x+1}$

6. $\sqrt[3]{352} = \sqrt[3]{7x^2+9}$

7. $5^x = 3125$

8. $9^x = 243$

$3 \; 3$; $9 \; 27$; $3 \; 3$; $9 \; 3$; $3 \; 3$

$3^{2x} = 3^5$

$\frac{\cancel{2}x}{\cancel{2}} = 5/2$

$\boxed{x = -2.5}$

9. $5^x = \frac{1}{125}$

make negative!

10. $4^x = \frac{1}{32}$

$2 \; 2$; $4 \; 8$; $2 \; 2$; $4 \; 2$; $2 \; 2$

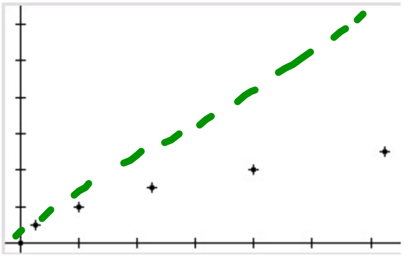
$2^{2x} = 2^{-5}$

$2x = -5$

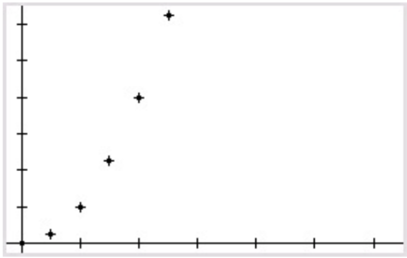
$\boxed{x = -2.5}$

11. $3 \cdot 2^x = 96$

12. Students were given a set of data to graph and were asked to work independently. After they had completed their graphs, each student shared his graph with his shoulder partner. When Ethan and Emma saw each other's graphs, they exclaimed together, "Your graph is wrong!" Neither graph is wrong. Explain what Ethan and Emma have done with their data.



Ethan's graph

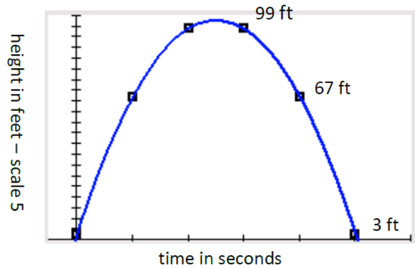


Emma's graph

13. Describe a sequence of transformations that would take Ethan's graph onto Emma's.

~~rotation~~ reflection across y=X
~~dilation~~ ~~translate~~

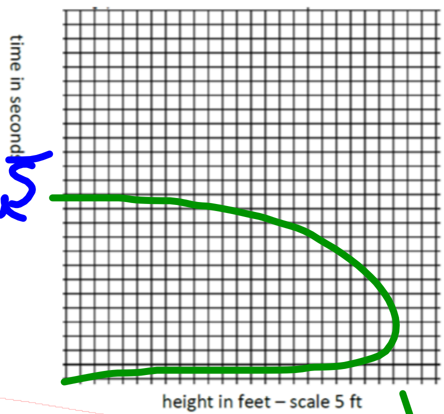
14. A baseball is hit upward from a height of 3 feet an initial velocity of 80 feet per second (about 55 mph). The graph shows the height of the ball at any second during its flight. Use the graph to answer the questions below.



a. Approximate the time that the ball is at its maximum height.
b. Approximate the time that the ball hits the ground.
c. At what time is the ball 67 feet above the ground?
d. Make a new graph that shows the time when the ball is at the given heights.

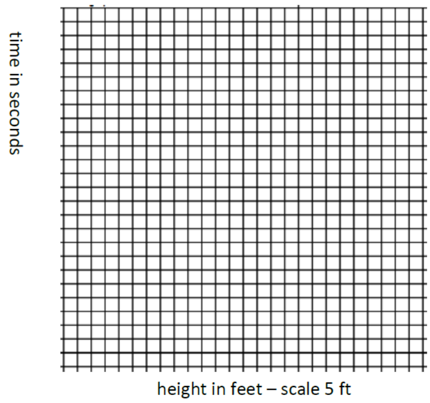
e. Is your new graph a function? Explain.

times



100ft

e. Is your new graph a function? Explain.



Go

Topic: Using function notation to evaluate a function.

The functions $f(x), g(x), and\ h(x)$ are defined below. Simplify your answers.

$f(x) = 3x$ $\qquad$ $g(x) = 10x + 4$ $\qquad$ $h(x) = x^2 - x$

19. $g(7)$ $\qquad$ 20. $g(-9)$ $\qquad$ 21. $g(s)$ $\qquad$ 22. $g(s - t)$

use h(x)
replace x with −9

FOIL $(s-t)(s-t)$

24. $h(7)$ $\qquad$ 25. $h(-9)$ $\qquad$ 26. $h(s)$ $\qquad$ 27. $h(s - t)$

$h(-9) = (-9)^2 - (-9)$

$81 + 9 = 90$

$= (s-t)^2 - (s-t)$

$s^2 - 2st + t^2 - s + t$

Notice that the notation *f(g(x))* is indicating that you replace *x* in *f(x)* with *g(x)*.

use f(x)
replace all x's with g(x).

Simplify the following.

28. *f(g(x))* $\qquad$ 29. *f(h(x))* $\qquad$ 30. *g(f(x))*

$= 3(10x+4)$

$= 30x + 12$

Mathematics Vision Project | MVP