3.5 The Expansion

A Develop Understanding Task

Polynomial functions have interesting characteristics. The degree of the polynomial not only tells us information about the end behavior of the function, it also tells us about the number of roots. In Secondary II, the Fundamental Theorem of Algebra was introduced while studying quadratic functions. The theorem states:

An n^{th} degree polynomial function has n roots.

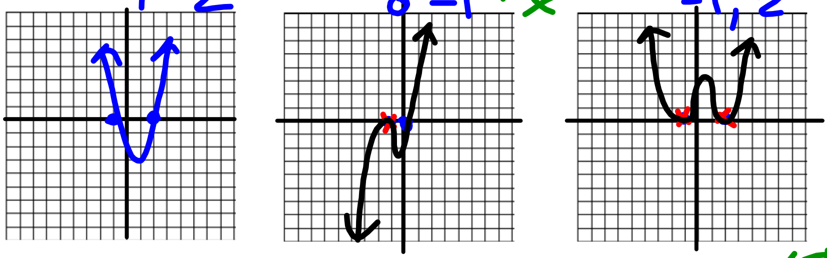
Warm UP

Find the zeros, shape and Graph!

1. $f(x) = (x+1)(x-2)$ — -1, 2

2. $f(x) = x(x+1)^2$ — $(x+0)$ — bounce — 0, -1 — $\sim x^3$

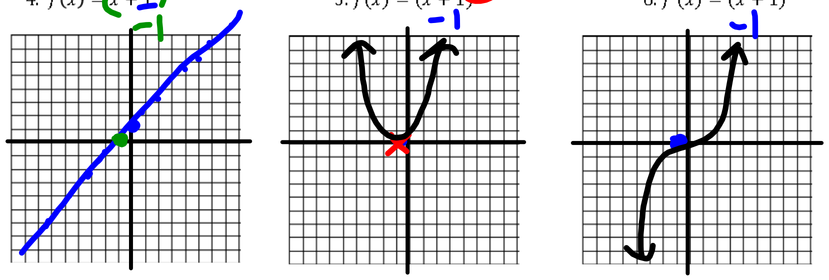
3. $f(x) = (x+1)^2(x-2)^2$ — bounce, bounce — -1, 2 — $\sim x^4$



4. $f(x) = (x+1)$ — -1

5. $f(x) = (x+1)^2$ — -1

6. $f(x) = (x+1)^3$ — pass — -1



7. Once you have made a conjecture about the graphs, confirm your solutions (using technology).

Intro to Imaginary Numbers

1. Starter: 3.5 Lesson graphs
2. Go over HW: 3.4 + Long Division
3. Quiz 3.4
4. Video Intro
5. Today's homework:
   a. Imaginary Number Practice WKS
   b. 3.5 HW (only Domain and Range)

3.5 The Expansion

A Develop Understanding Task

Desmos.com

Check using technology

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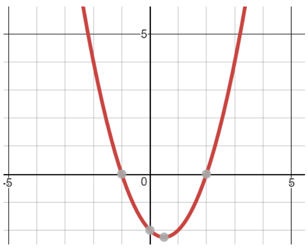
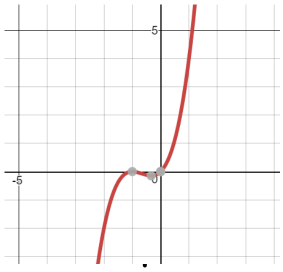
An n^{th} degree polynomial function has n roots.

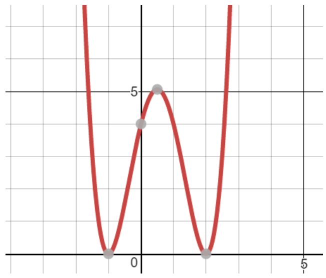
1. $f(x) = (x + 1)(x - 2)$



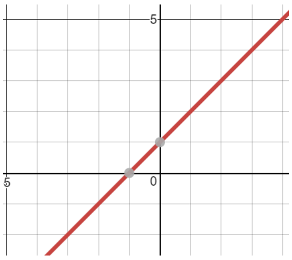
2. $f(x) = x(x + 1)^2$



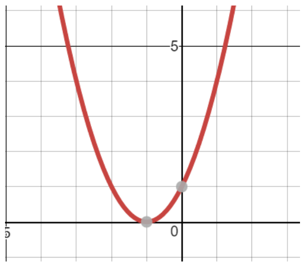
3. $f(x) = (x + 1)^2(x - 2)^2$



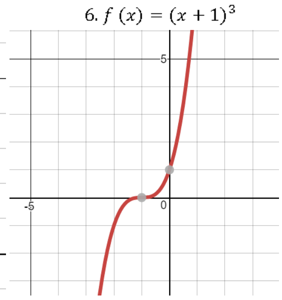
4. $f(x) = x + 1$



5. $f(x) = (x + 1)^2$



6. $f(x) = (x + 1)^3$



7. Once you have made a conjecture about the graphs, confirm your solutions (using technology).

matches graph #4

$f(x) = x + 1$

Expanded form:

____

Matches #5

$f(x) = (x + 1)^2$

Expanded form:

____

matches #6

$f(x) = (x + 1)^3$

Expanded form:

____

The coefficients of the polynomials are written in the table below:

$(x + 1)^0$	1
$(x + 1)^1$	1 1
$(x + 1)^2$	1 2 1
$(x + 1)^3$	1 3 3 1
$(x + 1)^4$	skip
$(x + 1)^5$	
$(x + 1)^6$	
$(x + 1)^n$	

10. Based on the pattern above, what do you think the coefficients would be for $(x + 1)^4$? How about $(x + 1)^5$?

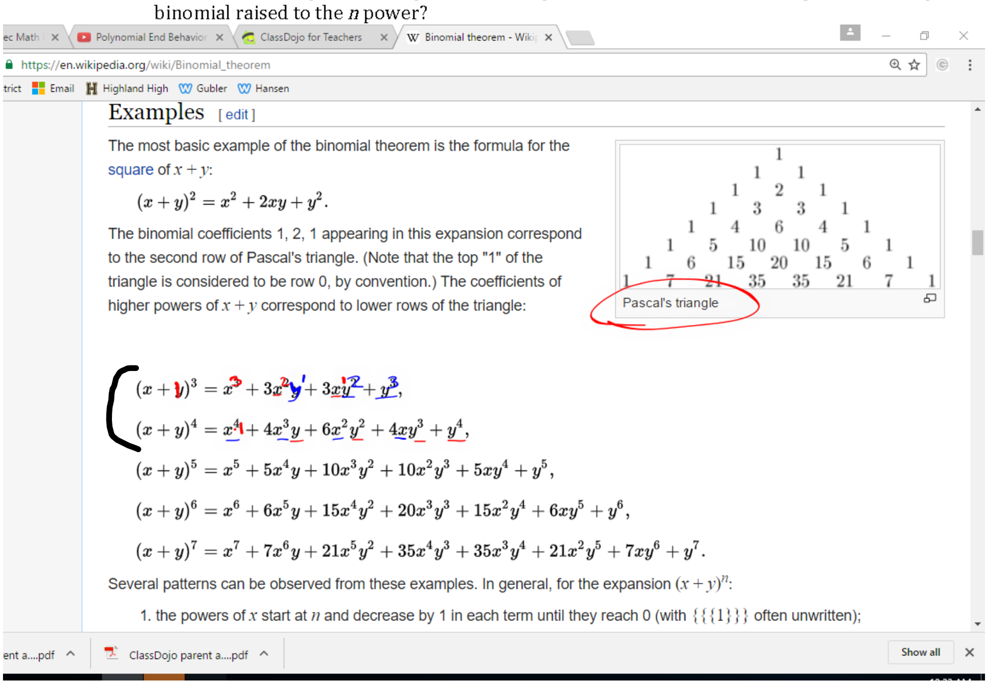
11. Describe how you find the coefficients of the binomial expansion for the 'next' expansion?

12. How would the coefficients change for the binomial: $(x + 2)$?

12. How would the coefficients change for the binomial: $(x+2)$?

13. How would the coefficients change for the binomial: $(x+y)$?

14. Extension: Do you have a system for how you could find the binomial expansion for *any* binomial raised to the n power?



ec Math | Polynomial End Behavior | ClassDojo for Teachers | Binomial theorem - Wiki

https://en.wikipedia.org/wiki/Binomial_theorem

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Examples [edit]

The most basic example of the binomial theorem is the formula for the square of $x+y$:

$$(x+y)^2 = x^2 + 2xy + y^2.$$

The binomial coefficients 1, 2, 1 appearing in this expansion correspond to the second row of Pascal's triangle. (Note that the top "1" of the triangle is considered to be row 0, by convention.) The coefficients of higher powers of $x+y$ correspond to lower rows of the triangle:

1
1 1
1 2 1
1 3 3 1
1 4 6 4 1
1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1

Pascal's triangle

$$(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3,$$
$$(x+y)^4 = x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^4,$$
$$(x+y)^5 = x^5 + 5x^4y + 10x^3y^2 + 10x^2y^3 + 5xy^4 + y^5,$$
$$(x+y)^6 = x^6 + 6x^5y + 15x^4y^2 + 20x^3y^3 + 15x^2y^4 + 6xy^5 + y^6,$$
$$(x+y)^7 = x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + y^7.$$

Several patterns can be observed from these examples. In general, for the expansion $(x+y)^n$:

1. the powers of x start at n and decrease by 1 in each term until they reach 0 (with {{{1}}} often unwritten);

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Name Polynomial Functions **3.5**

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Ready, Set, Go!

Ready

Topic: Describe the features of various functions

range

domain

*Find Domain and Range ONLY

1. $f(x) = x^2 - 16$

-16

D:(-∞,∞)

R:[-16,∞)

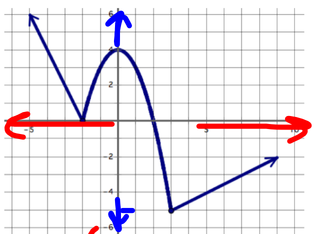
2. $f(n-1) = f(n) + 3;\ f(1) = 4$

linear!

3. $f(x) = \sqrt{x-3} + 1$

4. $f(x) = \log_2 x - 1$

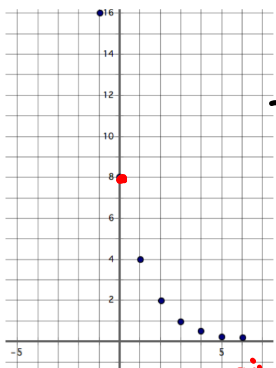
5.



D:(-∞,∞)

R:[-5,∞)

6.



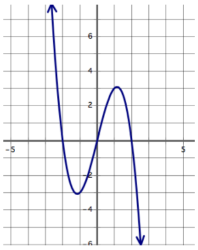
x	y
-1	16
0	8
1	4
2	2
3	1
4	1/2
5	1/4

÷2

domain

range

7.



Set

Topic: features of polynomial functions

Write key features of each function (intercepts, end behavior, and where the function is increasing/decreasing), then graph.

8. Equation: $f(x) = (x - 1)^2$ bounce

−1

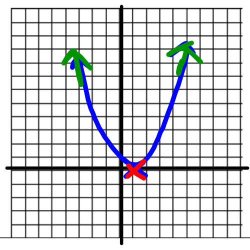
What I know about this function:

x=-1 , bounces
parabola

End behavior:

as $x \to -\infty$, $f(x) \to \infty$
as $x \to \infty$, $f(x) \to \infty$

Graph:



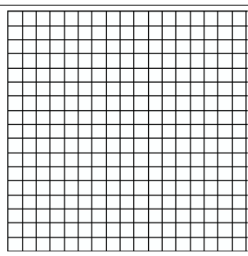
9. Equation: $h(x) = -x^2 + 1$

What I know about this function:

End behavior:

as $x \to -\infty$, $h(x) \to$ ____
as $x \to \infty$, $h(x) \to$ ____

Graph:



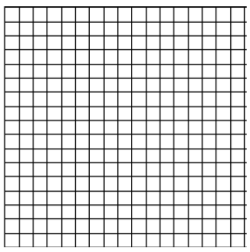
10. Equation: $h(x) = (x - 3)(x + 4)(x + 1)$

What I know about this function:

End behavior:

as $x \to -\infty$, $h(x) \to$ ____
as $x \to \infty$, $h(x) \to$ ____

Graph:



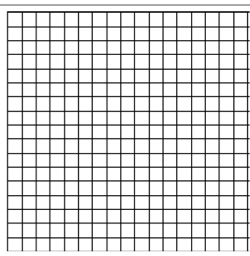
11. Equation: $f(x) = x^3$

What I know about this function:

End behavior:

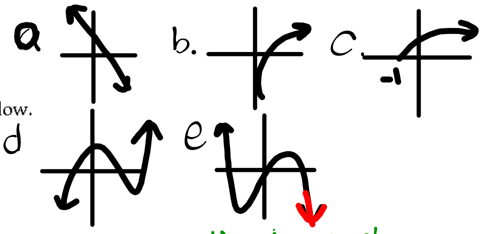
as $x \to -\infty$, $f(x) \to$ ____
as $x \to \infty$, $f(x) \to$ ____

Graph:



Name Polynomial Functions | 3.5

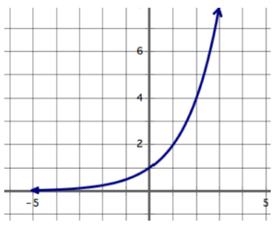
Go



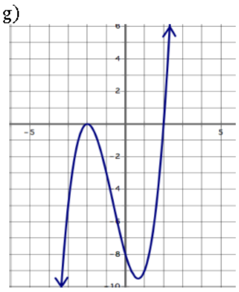
Topic: comparing functions in different forms.

Part II: Use functions a-h to answer the questions below.

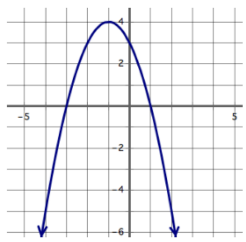
a) $f(x) = 3 - 2x$
b) $f(x) = log_2 x$
c) $f(x) = \sqrt{x+1}$
d) $f(x) = 3(x-1)(x+2)(x-4)$
e) $f(x) = -2x^3 + 2x^2 - x + 5$ don't know factors so just estimate graph
f)



g)



h)



12. Which function(s) do not have a domain of all real numbers?
13. Which function(s) do not have a range of all real numbers?
14. Which function(s) have exactly two x-intercepts?
15. Compare **a** and **c**: which has the greatest value as $x \to \infty$?
16. Compare **d** and **f**: which has the greatest value as $x \to \infty$?
17. Compare **f** and **g**: which has the greatest value as $x \to \infty$?
18. Compare **e** and **h**: which has the greatest value as $x \to \infty$?
19. Compare **g** and **h**: which has the highest relative maximum value?
20. Compare **b** and **f**: which has the greatest average rate of change from [15, 20]?