

7.8 Madison's Round Garden

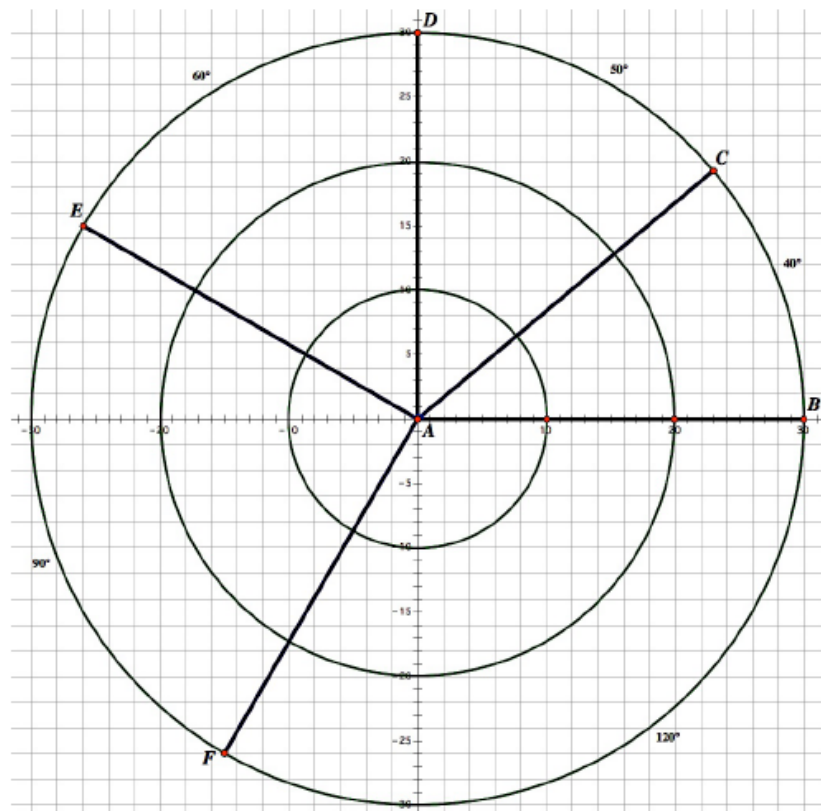
A Practice and Develop Understanding Task



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Last year Madison won the city's "Most Outstanding Garden" Award for her square garden. This year she plans to top that with her design for a beautiful round garden.

Madison's design starts with a sprinkler in the center, and concentric rings of colorful flowers surrounding the central sprinkler. Pavers will create both circular pathways and pathways that look like spokes on a wheel between the flowers. The sprinkler can be adjusted so it waters just the inner circle of flowers, or it can be adjusted so it waters the entire round garden. Consequently, flowers that need to be watered more frequently will be placed near the center of the garden, and those that need the least amount of water will be placed farthest from the center. The sectors of the garden will not all be the same size, since they need to accommodate different types of plants.



Here is Madison's design for her garden. The number of degrees in each sector has been marked.

1. Madison has only marked the degree measure on the arcs of the outermost ring of the garden. Determine the angle measure for the arcs on the inner and middle rings of the garden.
2. Madison needs to order pavers for the garden. She plans to vary the size and colors of the pavers in different parts of the garden. Consequently, she needs to know the lengths of different portions of the paths. Help her complete this table by calculating the missing arc lengths.

	Distance from Center	Arc Length				
		40° Sector	50° Sector	60° Sector	90° Sector	120° Sector
Inner Circle of Pavers	10 feet					
Middle Circle of Pavers	20 feet					
Outer Circle of Pavers	30 feet					

3. As Madison filled out the table she began to notice some interesting things. What did you notice?

4. One thing Madison noticed involved the ratio of the arc length to the radius of the circle. Complete this version of the table and state what you think Madison noticed.

	Distance from Center	Arc length / Radius				
		40° Sector	50° Sector	60° Sector	90° Sector	120° Sector
Inner Circle of Pavers	10 feet					
Middle Circle of Pavers	20 feet					
Outer Circle of Pavers	30 feet					

As Madison examined these numbers, she realized that they behave the same way that degree measurements behave—all arcs in the same sector have the same degree measurement, and all arcs in the same sector have the same value for the ratio of arc length to radius. This made her wonder if these new numbers could be used as a way of measuring angles just as degrees are used.

Later that evening Madison shared her discovery with her older sister Katelyn who is taking calculus at a local university. Katelyn told Madison that her new numbers for measuring angles in terms of the ratio of the arc length to the radius are known as *radians* and that they make the rules of calculus much easier than if angles are measured in degrees.

Madison learned so much from examining the arc length of the sectors of her garden that she decided to examine the areas of the sectors also.

- Complete this table for Madison by calculating the areas of the sectors for the different rings of the garden.

	Distance from Center	Area of Sector				
		40° Sector	50° Sector	60° Sector	90° Sector	120° Sector
Inner Circle of Pavers	10 feet					
Middle Circle of Pavers	20 feet					
Outer Circle of Pavers	30 feet					
Extended Circle of Pavers	40 feet					

- Do you notice anything interesting in this table?

SECONDARY MATH II // MODULE 7
 CIRCLES A GEOMETRIC PERSPECTIVE - 7.8

7.8

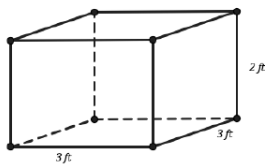
READY, SET, GO!	Name _____	Period _____	Date _____
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READY

Topic: Finding volume and surface area

Find the volume and surface area for the 3-dimensional shapes below.

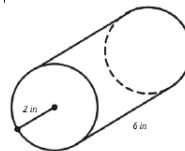
1.



a. Volume =

b. Surface Area =

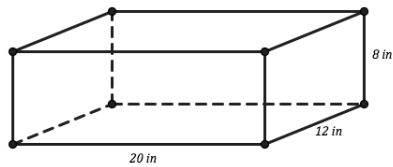
2.



a. Volume =

b. Surface Area =

3.



a. Volume =

b. Surface Area =

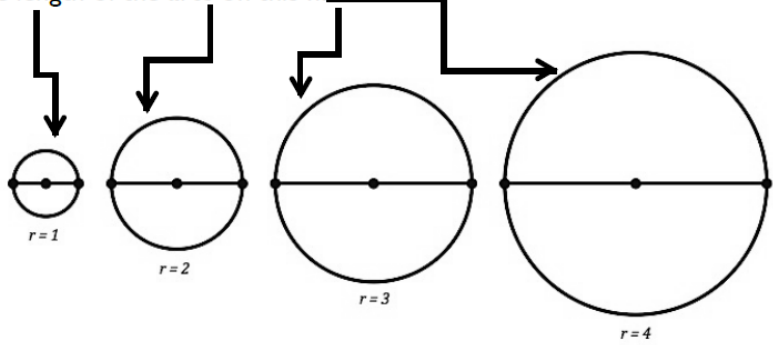
SET

Topic: Radians

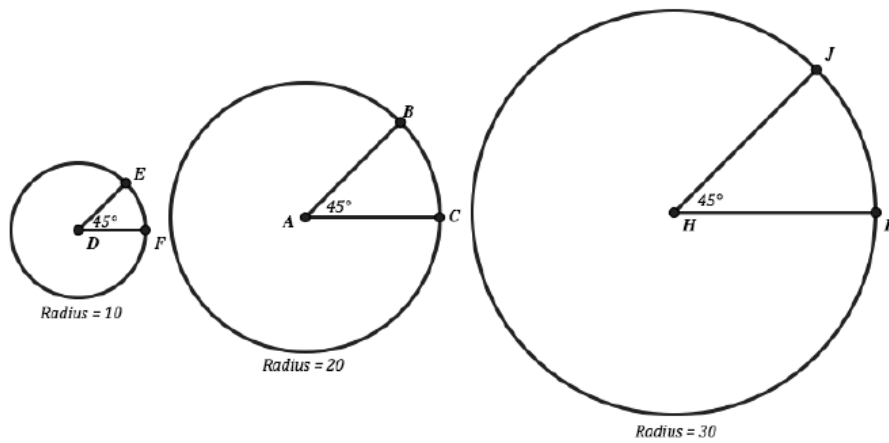
4. Below are circles of radius 1, 2, 3, and 4 units. Each of them has a diameter drawn that cuts them into two equal sectors. Find the arc length of one half of each of these circles. Then find the radian measure of the arc length for each one.

Radius	length of arc for half the circle	diagonal Radian measure of half the circle
1		
2		
3		
4		

Find the length of the arcs on this half



5. There are three circles below each with a different radius. The same size angle 45° has been used to create a sector in each circle. Fill in the table with the length of the arc measure for the sector, the radian measure and the area of the sector.



Review your Gold Wks

Radius	Length of arc	Radians	Area of sector
10			
20			
30			

6. Use the three circles in problem 5 to find the following ratios.

- a. \widehat{EF} to \widehat{BC}
- b. \widehat{BC} to \widehat{JK}
- c. \widehat{EF} to \widehat{JK}

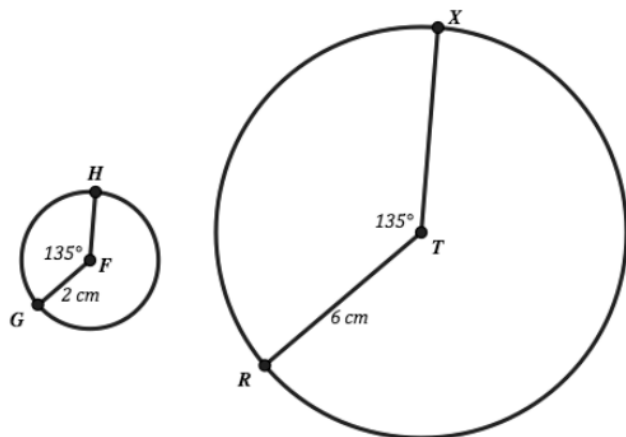
d. What do you notice about the ratios between the arc lengths?

7. Considering \widehat{EF} above in problem 5. (a) How many copies of this arc would be needed to be equal to the length of the entire circumference of circle D? (b) Would this be true for the other arcs and circles in the problem above? Why?

GO

Topic: Same angle different size sectors and arcs, accompanying ratios

Consider the sectors and arc lengths in the two circles below to answer the questions.



8. Find the arc length of arc GH.
9. Find the arc length of arc RX.
10. Find the area of the small sector in circle F.
11. Find the area of the small sector in circle T.
12. The Radian measure of the 135° sector in each circle.
- 13a. What is the ratio of arc GH to arc RX?
- b. What is the ratio of the areas of the two sectors?