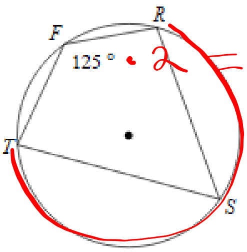
7.4 Warm-Up

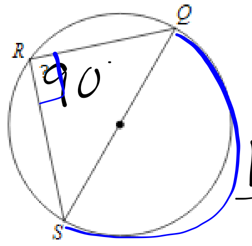
Find the measure of the arc or angle indicated.

1) Find $m\widehat{RST}$



$125 \cdot 2 = 250°$

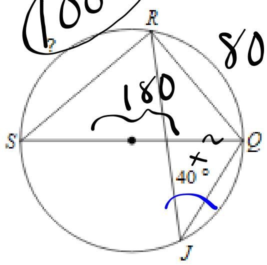
2)



$90°$

$\frac{180}{2}$

3)



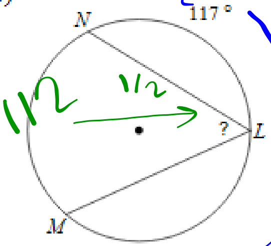
$100°$

80

180

x

4)



117
131
248

360
- 248
112

$\frac{112}{2}$

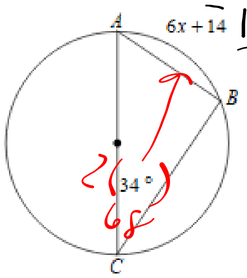
112

1/2

$56°$

Solve for *x*.

5)



2(34)
68

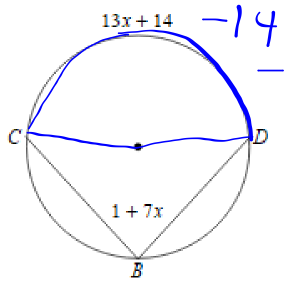
34
x 2
68
- 14
54

$54/6 = 9$

$x = 9$

$68 = 6x + 14$

6)



$2(1 + 7x) = 13x + 14$

$2 + 14x = 13x + 14$

$-14 \qquad\qquad -14$

$-12 + 14x = 13$

$-14x \qquad -14x$

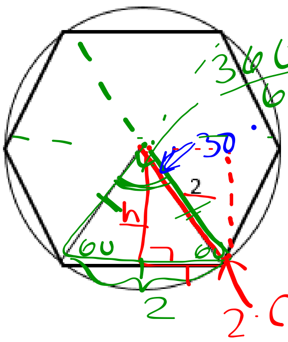
$-12 = -x$

$x = 12$

Sec 2 7.4 Planning the Gazebo

Objective: Determine the formula and evaluate to find the perimeter and area of regular polygons.

1. Find the Perimeter and Area of a Hexagon (n=6) with radius is 2.



$\frac{360}{6} = \frac{60}{2} = 30$

$A_\Delta = \cancel{2}\left(\frac{\text{base} \cdot \text{height}}{\cancel{2}}\right)$

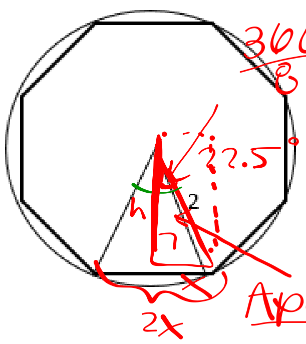
$A_\Delta = 1 \cdot 1.73 = 1.73$

$2 \cdot \cos 30 = \frac{h}{2}$ $h = 1.73$

perimeter $= 2 \cdot 6 = 12$

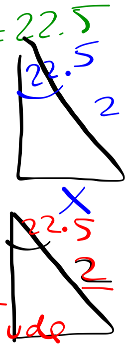
$\text{Area}_{\text{Hex}}: A_\Delta \cdot 6 = 1.73 \cdot 6 = \boxed{= 10.38 \text{ in}^2}$

2. Find the Perimeter and Area of an Octagon (n=8) with radius 2.



$\frac{360}{8} = \frac{45}{2} = 22.5$

Apothem
Altitude



$2 \cdot \sin 22.5 = \frac{x}{2}$

$x = .7654$

$2 \cdot \cos 22.5 = \frac{h}{2}$

$h = 1.8478$

perimeter: $2 \cdot x \cdot n$

$= 2(.7654) \cdot 8$

$\boxed{= 12.25 \text{ in}}$

Area: $\frac{\text{base} \cdot \text{height}}{} \cdot n$

$x \cdot h \cdot n$

$(.7654)(1.8478) \cdot 8$

$\boxed{= 11.31 \text{ in}^2}$

3. Write a general formula for Perimeter and Area for ANY Polygon with n sides.

$\theta = \frac{360}{2n}$

perimeter: $2 \cdot x \cdot n = 2(r \sin\theta) \cdot n = \boxed{2r(n \cdot \sin\theta)}$

Area $= x \cdot h \cdot n$

$(r \sin\theta)(r \cos\theta) \cdot n = \boxed{r^2(n \cdot \sin\theta \cdot \cos\theta)}$

4. Devise a formula to convert Perimeter to Area.

perimeter $= 2xn$ → want area $= x \cdot h \cdot n$

$\frac{(2xn) \cdot h}{2} = \frac{\text{perimeter}}{2} \cdot h = \text{Area}$

$\boxed{A = \frac{1}{2} p \cdot h}$

apothem

7.4 Planning the Gazebo

A Develop Understanding Task

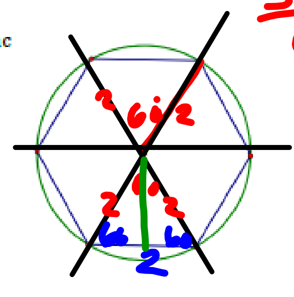
Find the Perimeter and Area of the Gazebo.

Zac is using his knowledge of geometry to design a gazebo for his family's back yard. The gazebo will be in the shape of a regular polygon. As part of his design, Zac will need to calculate several things so his parents can purchase the right amount of wood for the construction. For example, Zac will need to calculate the perimeter of the gazebo so he can order enough railing to surround it; he will need to calculate the area of the floor of the gazebo so he can order enough planks to lay it; and, he will need to calculate the surface area of the pyramid which forms the roof that will cover it. The problem is, his parents keep changing their minds about what shape they would like the gazebo to be—a hexagon, an octagon, a decagon, a dodecagon, or even some other type of *n*-gon.

From his work in Mathematics I with *Symmetries of Regular Polygons*, Zac knows that all regular polygons are cyclic—that is, every regular polygon can be ***inscribed*** in a circle. Zac is wondering if he can use this property of regular polygons to help him find their perimeter and area.

$\frac{360}{6} = 60^\circ$

For his first attempt at creating a scale drawing of the gazebo, Zac has inscribed a regular hexagon inside a circle with a radius of 2 inches. He is wondering if this is enough information to find the perimeter of this hexagon and the area it encloses.



Perimeter: $2(6) = 12$ in

Area$_{\triangle} = \frac{\text{base} \cdot \text{height}}{2}$

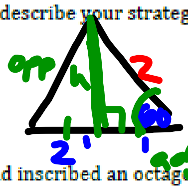
1. To get started with the task of finding the perimeter of this hexagon, Zac decides to write down what he already knows about this figure. Decide if you agree or disagree with each of his statements, and explain why. You will want to add features to the diagram to illustrate Zac's comments.

What Zac thinks he knows:	Do you agree or disagree? Explain why.
Two radii drawn to two consecutive vertices of the regular hexagon form a central angle whose measure can be found based on the rotational symmetry of the figure.	
The hexagon can be decomposed into 6 congruent isosceles triangles.	
The length of the altitudes of each of these 6 congruent triangles (the altitude drawn from the vertex of the triangle which is located at the center of the circle) can be found using trigonometry.	
The length of the sides of the triangle that form chords of the circle can be found using trigonometry.	

2. Based on what you and Zac know, find the perimeter of the hexagon that he inscribed in the circle with a radius of 2 inches. Illustrate and describe your strategy so someone else can follow it.

$p = 12$

3. Now find the area of the hexagon that Zac inscribed in the circle with a radius of 2 inches. Illustrate and describe your strategy so someone else can follow it.



$1 \cdot \tan 60 = \frac{h}{\cancel{1}}$ $h = 1.7$

$1^2 + h^2 = 2^2$

$h^2 = \sqrt{3} = h = \sqrt{3} = 1.7$

$A_\Delta = \frac{\cancel{2} \cdot (1.7)}{\cancel{2}}$

$A_\Delta = 1.7$

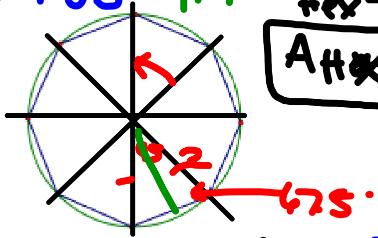
$A_{Hex} = 6(1.7)$

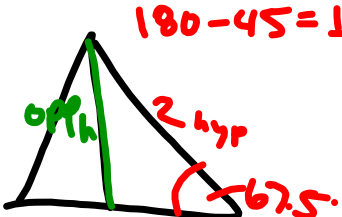
$\boxed{A_{Hex} = 10.2 in^2}$

4. What if Zac had inscribed an octagon inside the circle of radius 2 instead of a hexagon? Modify your strategy to find the perimeter and area of the octagon.

$\frac{360}{8} = 45$

$180 - 45 = \frac{135}{2} = 67.5$





base:

$2 \cdot \cos 67.5 = \frac{x}{\cancel{2}}$

$x = .77$

$base = 1.54$ double

height

$2 \cdot \sin 67.5 = \frac{h}{\cancel{2}}$

$h = 1.85$

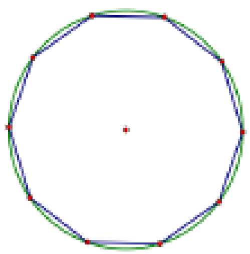
$A_\Delta = \frac{1.54(1.85)}{2}$

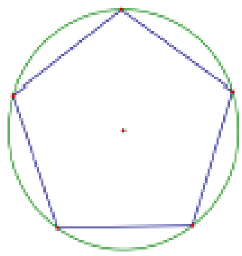
$A_\Delta = 1.42$

$A_{oct} = 1.42(8)$

$\boxed{= 11.4} in^2$

5. Modify your strategy to find the perimeter and area of any regular n-gon inscribed in a circle of any given radius.





SECONDARY MATH II // MODULE 7
CIRCLES A GEOMETRIC PERSPECTIVE - 7.4

7.4

READY, SET, GO! Name Period Date

READY

Topic: Radius and Area or Circumference $A = \pi r^2$ $C = 2\pi r$

Given the area or circumference or radius find the other two.

1.
Radius = 1 m
Area =
Circumference =

2.
Radius =
Area = 9π ft²
Circumference =

3.
Radius =
Area =
Circumference = 8π yds

4.
R =
A = 3.14 m²
C =

5.
R = 7 miles
A =
C =

6.
R = 40.5
A = $\pi(\ )^2 = 5152.9$
C = 81π inches $= 2\pi r$

$\frac{81\pi}{2\pi} = \frac{2\pi r}{2\pi}$

$40.5 = r$

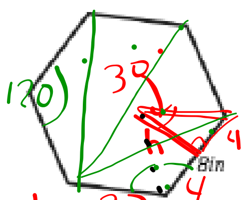
SET

Topic: Finding area and perimeter of regular polygons.

For each of the regular polygons find the measure of the interior angle, the perimeter and the area. $\frac{360}{6} = \frac{60}{2} = 30°$

$\frac{180(n-2)}{n}$

7.



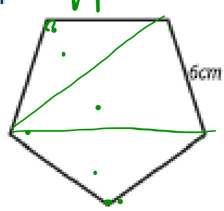
a. Measure of one interior angle: $\frac{180(4)}{6} = 120°$

b. Perimeter: $8 \cdot 6 = 48$

c. Area: $\frac{(4 \cdot 6.93) \cdot 6}{} \approx \boxed{166.3}$

$\tan 30 = \frac{4}{h}$ $h = 6.93$

8.

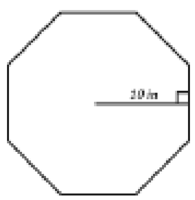


a. Measure of one interior angle:

b. Perimeter:

c. Area: 61.95

9.



a. Measure of one interior angle:

b. Perimeter:

c. Area:

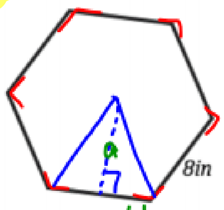
10. A regular polygon with 14 sides. And one side equal to 6 inches.

a. Measure of one interior angle:

b. Perimeter:

c. Area:

7.

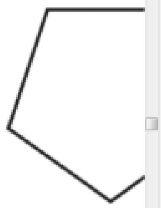


a. Measure of one interior angle:
$720° \div 6 = \boxed{120°}$

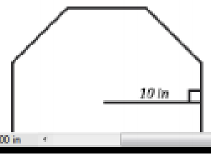
b. Perimeter:
$6 \cdot 8 = \boxed{48 \text{ in}}$

c. Area:
165.6in^2

8.



9.



a. Measure of one interior angle:

b. Perimeter:

10. A regular side equal to

a. Measure o

b. Perimeter

Another way to do #7

use perimeter ->area

formula.

a) Sum of Interior Angles in a Polygon:
$180(n-2)$
where $n = \#$ of sides

$\text{Sum} = 180(6-2)$
$= 180(4)$
$= 720°$

c) Area of a Regular Polygon:

$\frac{360}{6} = 60$

$A = \frac{1}{2} \cdot P \cdot a$

where P = perimeter and a = apothem

$A = \frac{1}{2}(48)(6.9)$

$\boxed{A = 165.6 \text{ in}^2}$

TOA

$a \cdot \tan 30 = \frac{4}{\cancel{a}} \cdot a$

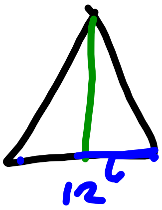
$\frac{a \cdot \cancel{\tan 30}}{\cancel{\tan 30}} = \frac{4}{\tan 30}$

$a = \frac{4}{\tan 30} = 6.9$

11. A 24-gon with sides equal to 12 meters.
a. Measure of one interior angle:

b. Perimeter:

c. Area:



12. A nonagon 9 with sides equal to 8 yards.
a. Measure of one interior angle:

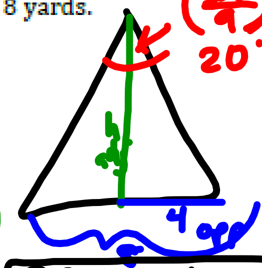
b. Perimeter: $9 \times 8 = 72$

c. Area: $\tan 20 = \frac{4}{h}$

$h = \frac{4}{\tan 20} = 10.99$

$A_{NON} = \left(\frac{8 \cdot 10.99}{2}\right)9 = \boxed{395.64 \text{ yds}^2}$

$\left(\frac{360}{9}\right)\frac{1}{2}$
20°



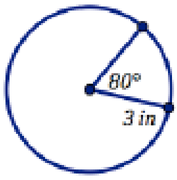
GO

Topic: Finding the area of a sector of a circle

If a circle is cut into four equal pieces then the area of one piece would clearly be one forth of the area of the entire circle. If a circle is cut into six equal pieces then the area of one of the pieces would be a sixth of the total area and so forth, for n equal pieces the area of one piece would be one nth of the total area. With this in mind consider the area of a sector that is one degree in size. A circle split into sectors that are all one degree in size would have 360 sectors. How could you find the area of just one of them?

Once you have one of them you could multiply it by any amount to find a sector of any number of degrees. Use this strategy to find the area of the sector of each circle on the next page. Use the example below to assist you.

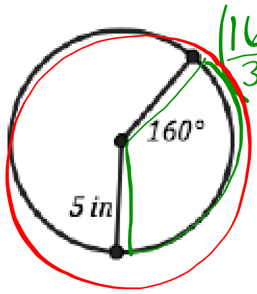
The area of the circle is 9π in^2
So, a sector of one degree would have area $\frac{9\pi}{360}$ in^2
And so the area of the sector with a central angle of 80°
Would be $(80)\frac{9\pi}{360}$ which simplifies to be 2π in^2.



Find the area of the sector indicated with the angle measure.

$\frac{\theta}{360}(\text{Area})$

13.

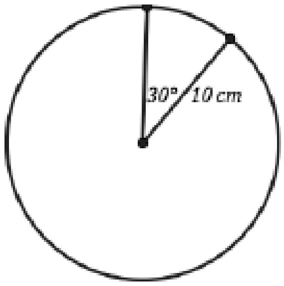


$\left(\frac{160}{360}\right)$ Area

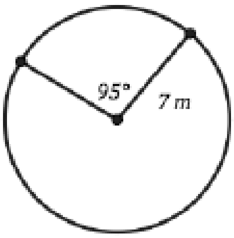
$\frac{4}{9}(\pi 5^2)$

$\boxed{= 34.9 \text{in}^2}$

14.



15.



16.