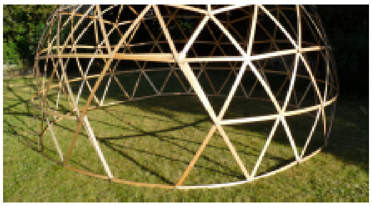
# 7.3 Cyclic Polygons

## *A Solidify Understanding Task*



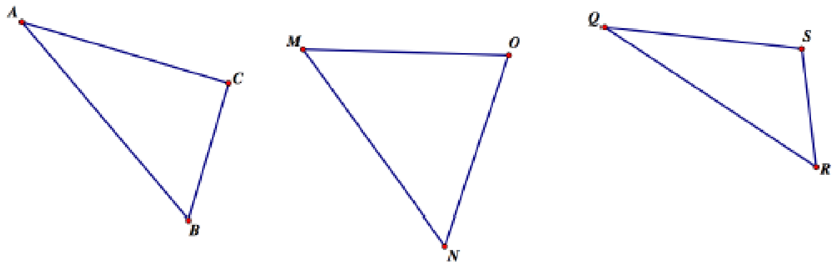


By definition, a cyclic polygon is a polygon that can be inscribed in a circle. That is, all of the vertices of the polygon lie on the same circle.

**Part 1**

In task *5.9 Centers of a Triangle* your work on Kara's notes and diagram should have convinced you that it is possible to locate a point that is equidistant from all three vertices of any triangle, and therefore all triangles are cyclic polygons.

1. Based on Kara's work, use a compass and straightedge to construct the circles that contain all three vertices in each of the following triangles.




Since each vertex of an inscribed triangle lies on the circle, each angle of the triangle is an inscribed angle. We know that the sum of the measures of the interior angles of the triangle is 180° and that the sum of the measures of the three intercepted arcs is 360°.

2. Using one of the diagrams of an inscribed triangle you created above, illustrate and explain why this last statement is true.

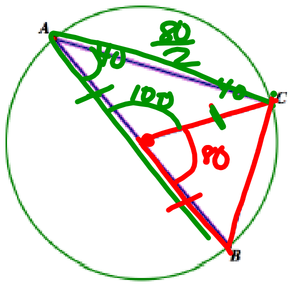
We know that the degree measure of an arc is, by definition, the same as the measure of the central angle formed by the radii that contain the endpoints of the arc. But how is the measure of an inscribed angle that intercepts this same arc related to the measure of the central angle and the intercepted arc? That is something useful to find out.

3. Using a protractor, find the measure of each arc represented on each circle diagram above. Then find the measure of each corresponding inscribed angle. Make a conjecture based on this data.

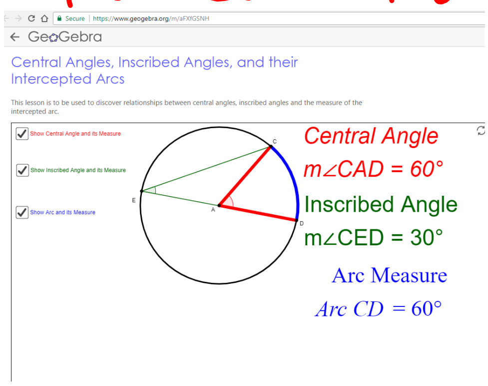
   My conjecture about the measure of an inscribed angle:

The three circle diagrams you created above have been reproduced below. One inscribed angle has been bolded in each triangle. A diameter of the circle has also been added to each diagram as an auxiliary line segment, as well as some additional line segments that will assist in writing proofs about the inscribed angles. Three cases are illustrated: case 1, where the diameter is a side of the inscribed angle; case 2, where the diameter lies in the interior of the inscribed angle; and case 3, where the diameter lies in the exterior of the inscribed angle. In each diagram, prove your conjecture about the measure of an inscribed angle for the inscribed angle shown in bold.

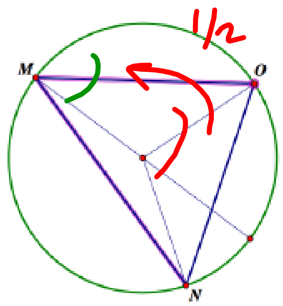
Case 1: [Hint: Look for isosceles triangles and an external angle of a triangle.]

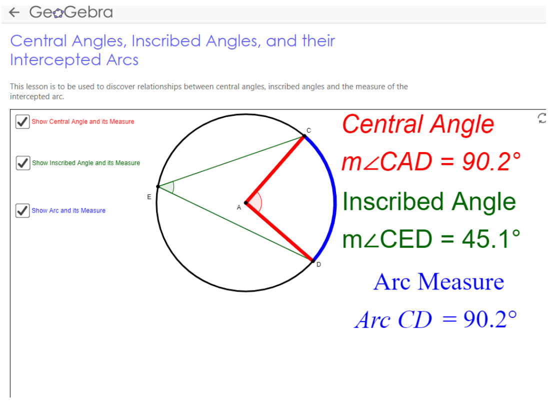



the inscribed angle is 1/2 of the central angle



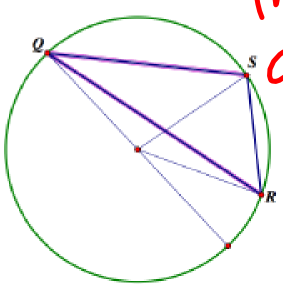
Case 2: [Hint: Can you see case 1 in case 2?]

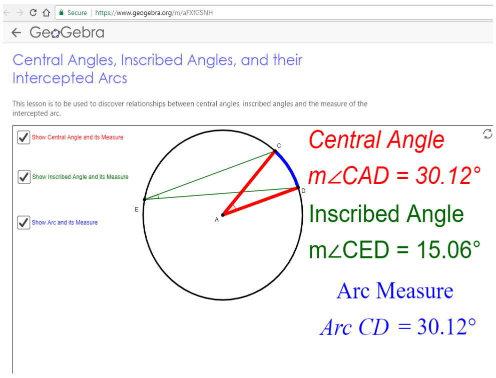



Case 3:

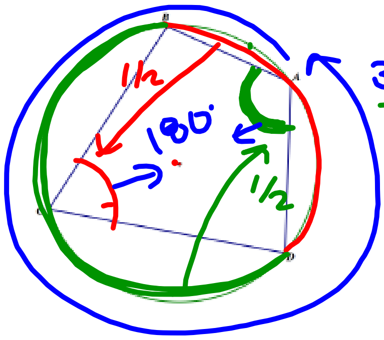
inscribed is always 1/2 of central



Proof of my conjecture:

(How might you use the following diagram to assist you in your proof?)



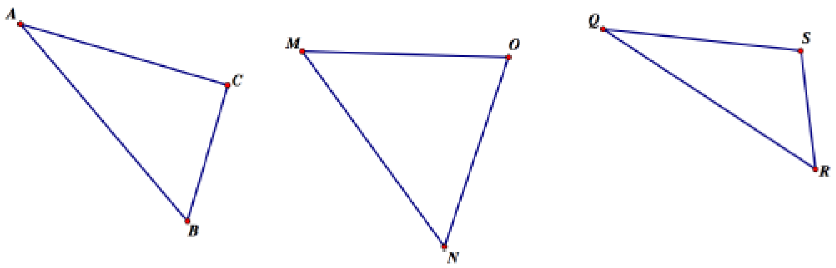

$\frac{360}{2} = 180°$

opposite angles in a quad. are supplementary (+180)

**Part 3**

In task *5.8 Centers of a Triangle*, your work on Kolton's notes and diagram should have convinced you that it is possible to locate a point that is equidistant from all three sides of a triangle, and therefore a circle can be inscribed inside every triangle.

6. Based on Kolton's work, use a compass and straightedge to construct the circles that can be inscribed in each of the following triangles. Once you have located the center of the inscribed circle, how do you determine where the points of tangency between the circle and the sides of the triangle are located?



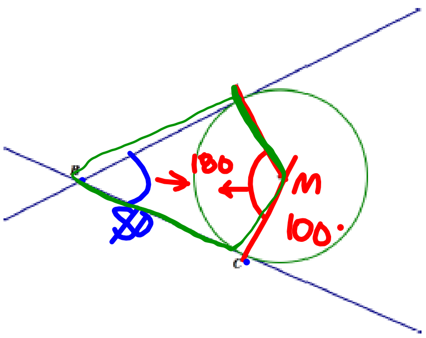
7. Angles formed by lines that are tangent to a circle are called circumscribed angles. Use dynamic geometry software to experiment with the measures of circumscribed angles relative to the arcs they intercept. Make a conjecture about the measures of the circumscribed angles. Then prove your conjecture using what you know about inscribed angles.

   My conjecture about the measures of circumscribed angles:

   $\angle ABC + \angle AMC = 180^\circ$

   $80 + 100^\circ = 180^\circ$

   Proof of my conjecture:




because opposite angles of a quadrilateral are supplementary.

8. Based on your work in this task and the previous task, describe a procedure for constructing a tangent line to a circle through a given point outside the circle.

SECONDARY MATH II // MODULE 7
CIRCLES A GEOMETRIC PERSPECTIVE - 7.3

7.3

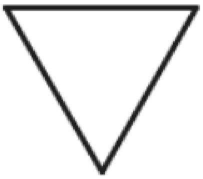
READY, SET, GO! Name Period Date

## READY

Topic: Symmetry and Trigonometric Ratios

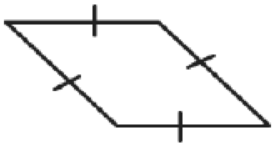
**Determine the** angles of rotational symmetry **and the number of** lines of reflective symmetry **for each of the polygons below.**

1. Equilateral Triangle

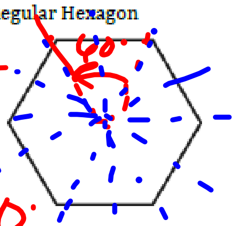


2. Rectangle

3. Rhombus



4. Regular Hexagon

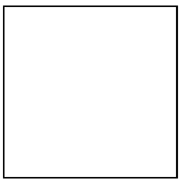


$\frac{360}{6} = 60°$

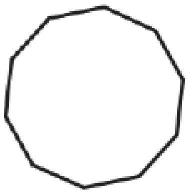
$60°$

6 lines of ref.

5. Square

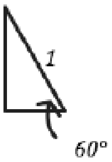


6. Regular Decagon

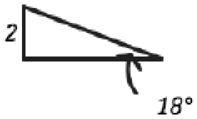


**Solve each right triangle, give the missing angles and sides.**

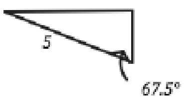
7.



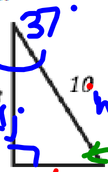

8.




9.




10.




37°

hyp

adj

6

$\cos^{-1}\left(\frac{8}{10}\right) = 37°$

$90 - 37 = 53°$

$8^2 + b^2 = 10^2$

$b = 6$

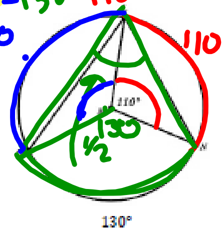
**SET**

Topic: Angles and how they connect with arcs.

**Find the value of the angle or the intercepted arc indicated in each figure below.**

11. ⊙ $M$ with $m\angle LMN = 110°$

$360 - 130 - 110$
$= 120$

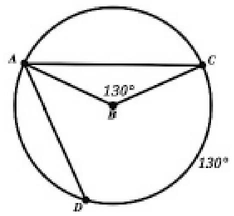



a. $m\widehat{LN} =$ 110

b. $m\angle OLN =$ 65

c. $m\widehat{OL} =$ 120°

12. ⊙ $B$ with $m\angle ABC = 130°$

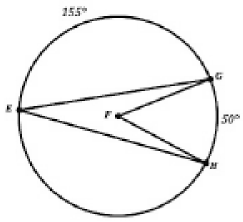



a. $m\widehat{AC} =$

b. $m\angle CAD =$

c. $m\widehat{DA} =$

13. ⊙ $F$




a. $m\widehat{EG} =$

b. $m\widehat{EHG} =$

$m\angle GEH =$

$m\angle GFH =$

14. ⊙ $M$ with diameter $\overline{NK}$

NL=12 and KL= 5

a. $NK =$ 13

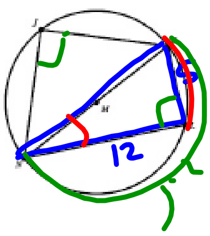
b. $m\widehat{NLK} =$ 180°

c. $m\widehat{NJK} =$ 180°

d. $m\angle NLK =$ 90°

e. $m\widehat{KL} =$ 45

f. $m\widehat{NL} =$ 135




$+12^2$

$\tan^{-1}\left(\frac{5}{12}\right) = 22.6$
$\times 2$
$45.24$

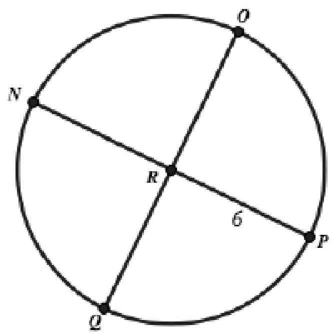
15. How can a triangle be used to show the connection between an inscribed angle and the angle measure of the arc it intercepts? What is true about the angle measures in any triangle? What is true about the arc measure for an entire circle?

## GO

Topic: Finding lengths of arcs

**Use what you know about finding circumference, C=2πr and Area, A=$\pi r^2$ for circles to find the indicated distances and areas below.**

16. ⊙ *R* is cut by two diameters that are perpendicular to each other.

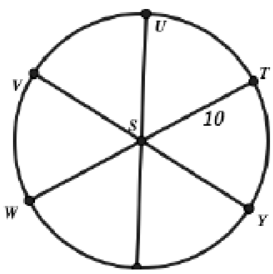



a. Find the distance to walk along arc NQ

b. Find the area inside one of the four sectors

c. Find the distance to walk along the following path: Start at point P and go to R then to Q and over to N then back to P.

17. ⊙ *S* is cut by three diameters that create equal angles at the center of the circle.




a. Find the distance to walk along arc UT

b. Find the area inside one of the six sectors

c. Find the distance to walk along the following path: Start at point U go to S then to V then to W followed by X and then back to U.