SECONDARY MATH II // MODULE 7
CIRCLES: A GEOMETRIC PERSPECTIVE – 7.11

7.11 Footprints in the Sand

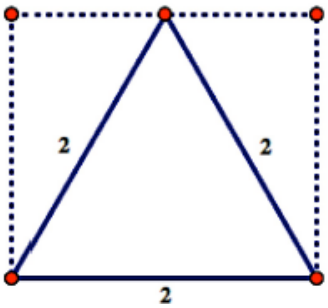
A Solidify Understanding Task

Benji, Chau and Kassandra are discussing the various three-dimensional shapes they plan to include in their sand castles. They are wondering how to calculate the volume of some of the shapes they want to include. Chau wants to include prisms with equilateral trianglular bases and Kassandra wants to include prisms with regular hexagonal bases. Benji only knows that the formula for a rectangular prism is $L \times W \times H$, and so he is trying to figure out how the shape of the base affects the volume of the prism.

Benji has heard his father, who is an architect, talk about the footprint of a building, which refers to the shape and area that a building will occupy on a plot of land. Benji likes the term, and wonders if thinking about the footprint of the prisms Chau and Kassandra want to include in the sand castles will help him figure out their volumes.

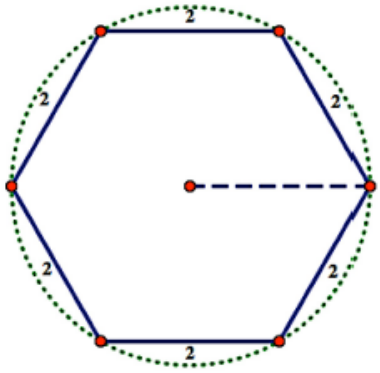
Chau wants to include a triangular prism with bases that are equilateral triangles, 2 inches on a side and 10 inches tall. Benji is examining the footprint of Chau's prism, inscribed in a rectangle.

1. Develop a strategy for finding the volume of Chau's prism using this drawing that Benji created to help him visualize the footprint of Chau's triangular prism.



Kassandra wants to include a hexagonal prism with bases that are regular hexagons, 2 inches on a side, and the prism is 10 inches tall. Benji is examining the footprint of Kassandra's prism, inscribed in a circle.

2. Develop a strategy for finding the volume of Kassandra's prism, using this drawing that Benji created to help him visualize the footprint of Kassandra's hexagonal prism.

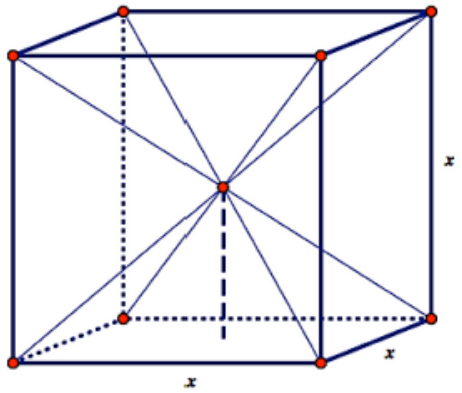


3. Describe a general procedure for finding the volume of a prism when you are given a description and dimensions of the bases of the prism.

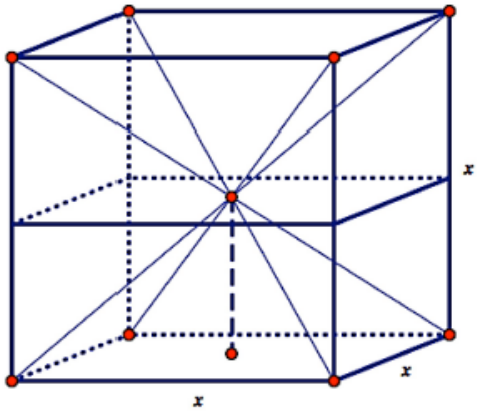
Benji has described his strategy for finding the volume of any prism to Chau and Kassandra. They are both excited by his findings, but Kassandra has another question. "I have always wondered why the volume of pyramids or cones is always $^1/_3$ of the volume of the prism or cylinder with the same base and height."

Chau replies, "I'm not sure why it is true in general, but I think I can explain it for a square pyramid whose height is ½ of the side length of the square that forms the base." Chau quickly sketches the following cube with all four of its diagonals. She has labeled the length of each edge of the cube as x inches.

4. The diagonals divide the cube into 6 congruent pyramids. (Each face of the cube is the base of one of the pyramids.) How is the volume of each of these pyramids related to the volume of the cube? Use Chau's drawing and the relationship between the volumes of the cube and the pyramids to derive a formula for the volume of one of the pyramids in terms of x.



5. The pyramid in question 4 does not have the same height as the cube. Find the volume of the rectangular prism that has the same base and height as one of the pyramids.



6. How is the volume of the pyramid described in question 4 related to the volume of the rectangular prism described in question 5?

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7.11

READY, SET, GO! Name Period Date

READY

Topic: Using the distance formula.

In Math 1 you should have developed the distance formula $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.
Find the exact distance between the two given points.

1. $A\ (3, -7)$ $B\ (-9, -2)$

2. $C\ (122, 367)$ $D\ (106, 304)$

3. $E\ (-231, -29)$ $F\ (-220, 31)$

4. $G\ (2, -4)$ $H\ (-1, 3)$

5. $K\ (1, 0)$ $L\ (0, \sqrt{2})$

6. $M\ (-11, 7)$ $P\ (-6, \sqrt{6})$

$x_1\ y_1\ x_2\ y_2$

$\sqrt{(-6 + 11)^2 + (\sqrt{6} - 7)^2}$ $(\sqrt{6} - 7)$

$(5)^2$ $6 - 14\sqrt{6} + 49$

$\sqrt{25 + 55 - 14\sqrt{6}}$

$\sqrt{80 - 14\sqrt{6}}$ this is the exact answer

~ 6.76

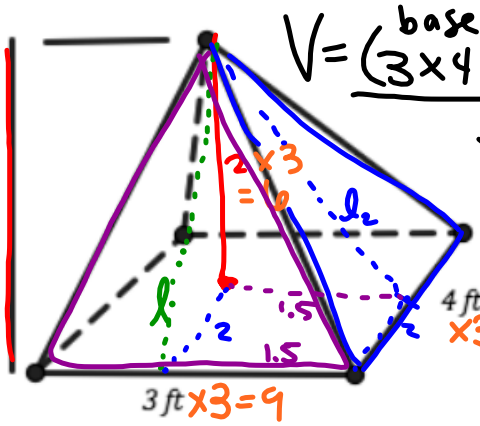
SET

Topic: Finding surface area and volume for similar solids.

Find the surface area and volume of each pyramid or cone.

$V_{pyramid} = \frac{1}{3}$

7. ℓ = slant height



$V = \frac{(3 \times 4)}{3} \frac{\text{base}}{2} \frac{\text{height}}{8}$ $8\,ft^3$

$\frac{(9 \cdot 12)6}{3} = 216\,ft^3$ ($\times 3^3$)

$2 \times 3 = 6$

$4\,ft \times 3 = 12$

$3\,ft \times 3 = 9$

$\ell = \sqrt{2^2 + 1.5^2} = 2.83$

$\ell_2 = \sqrt{2^2 + 1.5^2}$

$\ell_2 = 2.5$

8. A pyramid that is similar to the pyramid in number 7 but scaled up by a factor of 3.

Surface Area:

$2(\triangle) + 2(\triangle) + \text{base}$

$2\left(\frac{3 \times 2.83}{2}\right) + 2\left(\frac{4 \times 2.5}{2}\right) + (3 \times 4)$

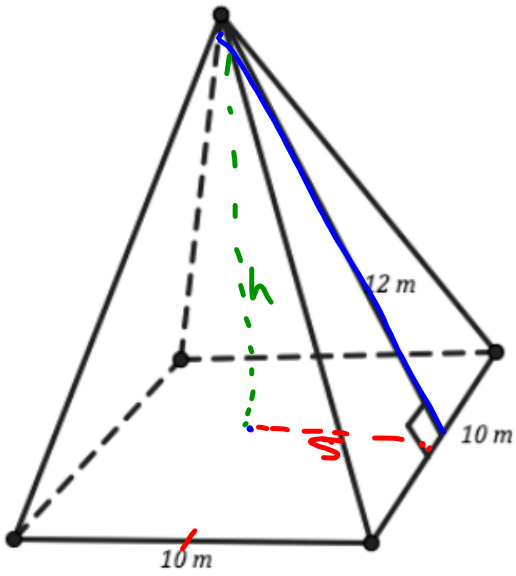
$SA = 30.49\,ft^2$ original

Scale factor of 3, $\times 3^2$

$(30.49) \times 3^2 = 274.41\,ft$

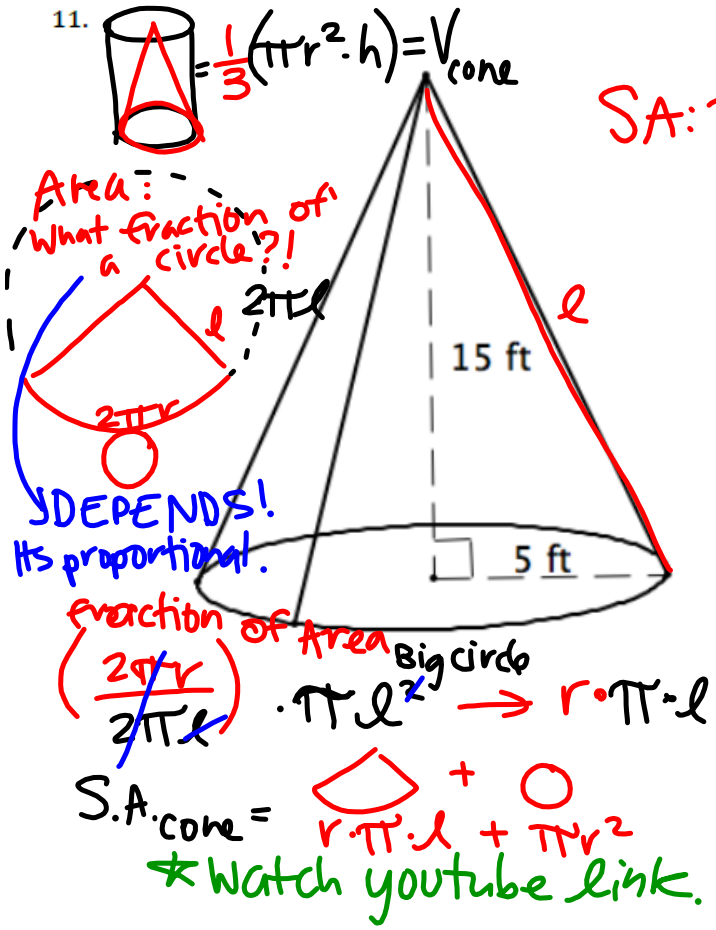
3x's Bigger

9.



10. A pyramid that is similar to the pyramid in number 9 but scaled up by a factor of 5.

11.



12. A cone that is similar to the one at the left that has been scaled up by a factor of 4.

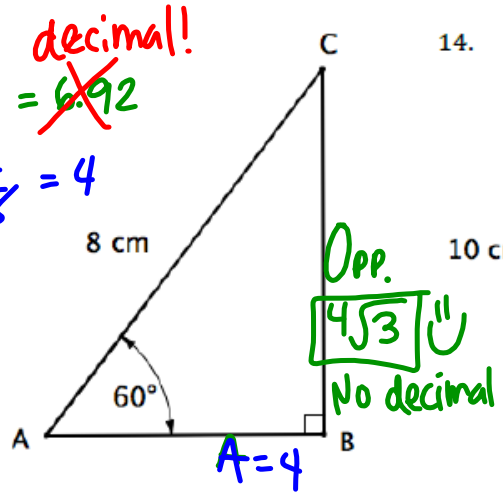
$SA: \pi r \cdot \ell + \pi r^2$

GO Topic: Finding the missing measures in a triangle.

Try to get EXACT answers. If you get a decimal, find a different side, then use pythagorean T. This will help you see the pattern!

Find the missing angles and sides in each triangle.

13.



decimal!

$8 \cdot \sin 60 = \frac{O}{8} = 6.92$

$8 \cdot \cos 60 = \frac{A}{8} = 4$

pythag:

$8^2 - 4^2 = O^2$

$\sqrt{48} = O$

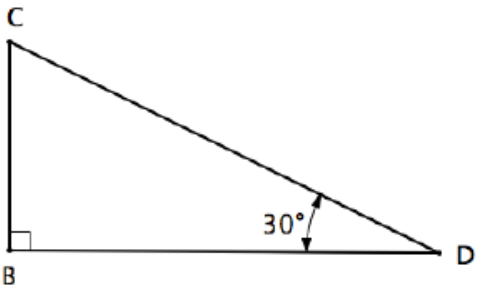
$4\sqrt{3}$

Opp. $4\sqrt{3}$ ft

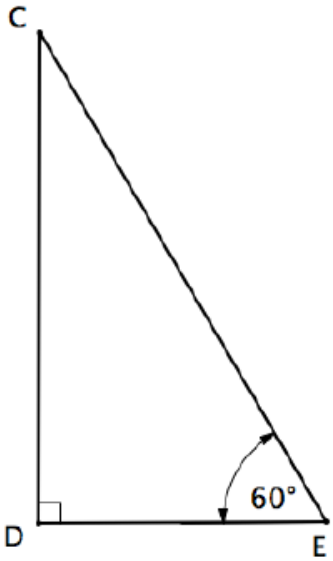
No decimal

A = 4

14.



15.



16. Be sure to find $m\angle 1$ *and* $m\angle 2$ *and* DG