SECONDARY MATH II // MODULE 7
CIRCLES: A GEOMETRIC PERSPECTIVE – 7.10

7.10 Sand Castles

A Solidify Understanding Task

Benji, Chau and Kassandra plan to enter a sand castle building contest being sponsored by a local radio station. The winning team gets a private beach party at a local resort for all of their friends. To be selected for the competition, the team has to submit a drawing of their castle and verification that the design fits within the rules.

The three friends actually plan to build three identical castles, each one twice as big as the previous one. They hope that replicating the same design three times—while paying attention to the tiniest little details—will impress the judges with their creativity and sand sculpting skill.

Benji is puzzling over a couple of questions on the application. They sound like math questions, and he wants Chau and Kassandra to make sure that he answers them correctly.

Please provide the following information about your sand sculpture:

- ***What is the total area of the footprint of your planned sand sculpture?***

[This information will allow the planning committee to locate sand sculptures so the viewing public will have easy access to all sculptures. Remember that the total area occupied by your sculpture cannot exceed 50 sq. ft.]

- ***What is the total volume of sand required to build your sand sculpture?***

[We will provide clean, sifted sand for each team so we will not be liable for any debris or harmful substances that can be present in beach sand.]

I certify that the above information is correct.

Signature of team leader: _____ ***date:*** _____

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The friends have only designed one of the castles, since the others will be scaled up versions of this one, each one being "twice as big".

After studying the diagram Benji said, "I calculated the area of the footprint of the smallest castle to be 2.5 sq. ft., so the next one will occupy 5 sq. ft., and the largest 10 sq. ft. That's a total of 17.5 sq. ft. Well within the limits."

1. What do you think of Benji's comment? Design a couple of possible "footprints" for a sand castle that will occupy 2.5 square units of area. Then scale each design up so it is "twice as big", and calculate the area. What do you notice?

2. Imagine stacking cubes on your sand castle "footprints" to create a simple 3-D sculpture. Then scale up each design so it is "twice as big" and calculate the volume. What do you notice?

3. How did you interpret the phrase "twice as big" in your work on questions 1 and 2? Is your interpretation the same as Benji's?

4. To avoid confusion, it would be more appropriate for Benji and his friends to say they are going to "scale up" their initial sand castle by a factor of 2. If the "footprint" of a sand castle occupies 2.5 sq. ft., is it possible to calculate the area occupied by a sand castle that has been enlarged by a scale factor of 2, or is the area of the enlarged shape dependent upon the shape of the original figure? That is, do triangles, parallelograms, pentagons, etc. all scale up in the same way? Write a convincing argument explaining why or why not?

5. What happens to the perimeter of the "footprint" of your sand castle when it is scaled-up by a factor of 2?

6. Suppose your sand castle "footprint" was cut out of a piece of Styrofoam that is one-inch thick. What happens to the volume when this "3-D footprint" is scaled up by a factor of 2?

7. The plans for the smallest sand castle include a rectangular prism that is 5 inches high and has a square base with a side length of 2 inches.

   a. What is the volume of sand required to make this prism in the smallest sand castle?

   b. What is the volume of sand required to make this prism in the middle-sized sand castle?

   c. What is the volume of sand required to make this prism in the largest sand castle?

   d. What is the perimeter of each of the squares that form the bases of each of the three different prisms in each of the three different sand castles?

   e. What is the total surface area of each of the rectangular prisms to be used in constructing each of the three sand castles? (This information is needed to construct nets for the molds that will be used to create the prisms.)

8. Chau and Kassandra's plans for the smallest sand castle include columns in the shape of cylinders with the base being a circle with a radius of 1 inch. The height of the column is 12 inches.

   a. What is the volume of sand required to make each of these columns in the smallest sand castle?

   b. What is the volume of sand required to make this column in the middle-sized sand castle?

   c. What is the volume of sand required to make this column in the largest sand castle?

   d. What is the circumference of each of the circles that form the circular bases of each of the three different columns in the three different sand castles?

   e. What is the total surface area of the cylinders—including the two circular bases and the rectangle that wraps around to form the cylinder—in each of the three sand castles? (This information is needed to construct the molds in which wet sand will be poured to create the columns.)

CIRCLES A GEOMETRIC PERSPECTIVE – 7.10

7.10

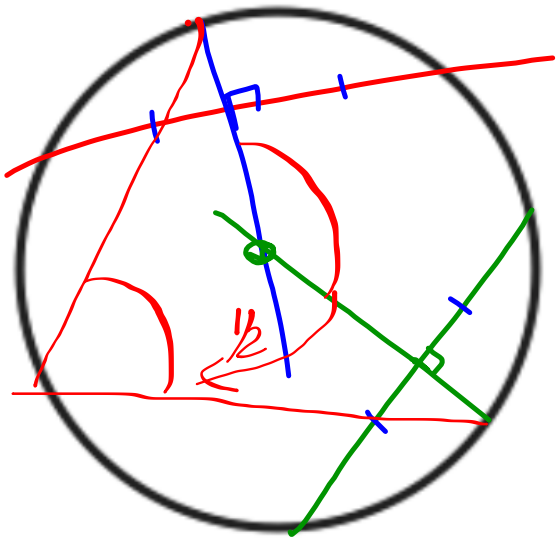
READY, SET, GO! Name Period Date

Ready

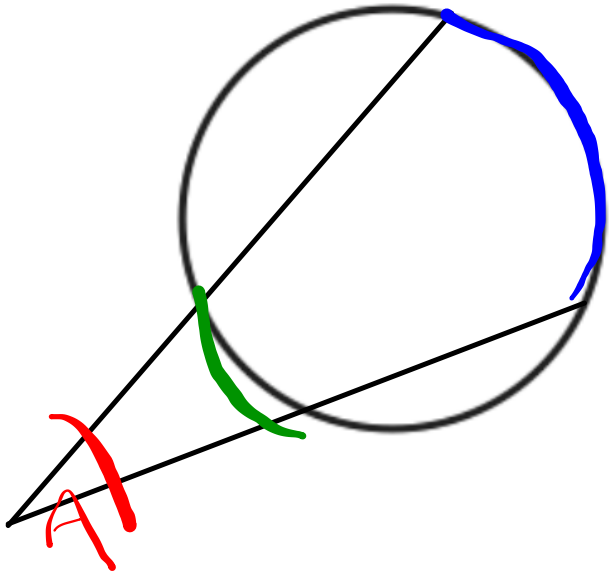
Topic: Finding the center of a circle.

Locate the center of each circle below. (Hint: Use chords of the circle to pinpoint the center.)

1.



2.



3. Justify your work for finding the center of the circles above. Why does it work? Why does it pinpoint the center of the circle?

4. In circle 1, draw a central angle and an inscribed angle that cuts the same arc as the central angle. What is the relationship between the measure of a central angle and its corresponding inscribed angle?

1/2

5. In circle 2, draw a central angle and a circumscribed angle that cuts the same arc as the central angle. What is the relationship between the measure of a central angle and its corresponding circumscribed angle?

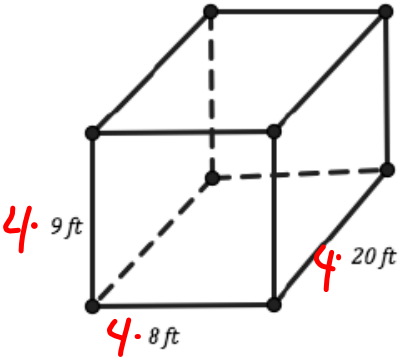
$A = \frac{1}{2}(far - near)$

Set

Topic: Finding surface area and volume of cylinders and rectangular prisms.

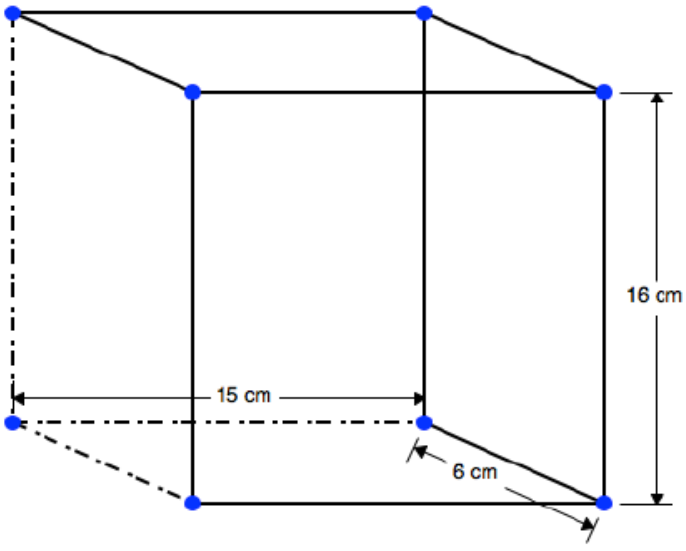
Find the surface area and volume of each rectangular prism.

6.



7. A prism similar to the one on the left that has been enlarged by a factor of 4.

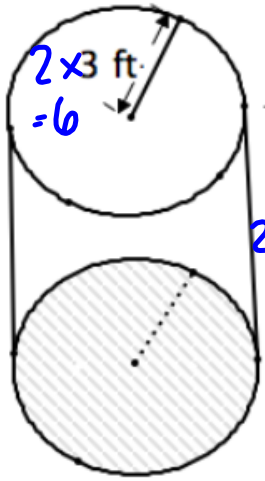
8.



9. A prism similar to the one on the left that has been enlarged by a factor of 3.

Find the surface area and volume of each cylinder.

10.



$SA = 0 + 0 + \square h$ (perimeter $2\pi r$)

$SA = 2(\pi r^2) + (2\pi r \times h)$

$2(\pi 3^2) + (2\pi 3 \times 8)$

$56.55 + 150.8$

$SA = 207.3$

Volume: (base πr^2) · height

$(\pi 3^2) \times 8$

$V = 226.2$

11. A cylinder similar to the one on the left that has been enlarged by a factor of 2.

$\times 2^2$

$2(\pi 6^2) + (2\pi 6 \times 16)$

$226.2 + 603.2$

$SA. = 829.38$

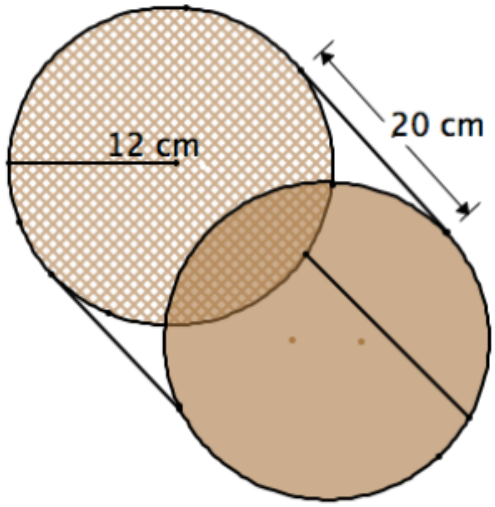
$= (\pi 6^2) \times 16$

$\times 2^3$

$V = 1809.6$

Area is squared! Volume is cubed!

12.



13. A cylinder similar to the one on the left that has been reduced by a factor of ½.

Starter 7.10 "go"

Go
Topic: Finding Centers of Rotation

Find the measure that is missing, either degrees or radians given the other measure.

14. 120° = Radians

15. 270° = Radians

16. 210° = Radians

17. $\frac{3\pi}{4}$ Radians = Degrees

18. 4.7 Radians = Degrees

19. $\frac{\pi}{6}$ Radians = Degrees

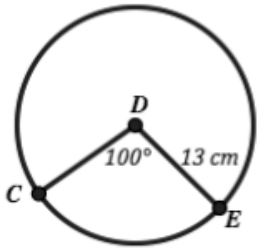
20. 300° = Radians

21. 180° = Radians

22. 360° = Radians

Find the area of each sector.

23.



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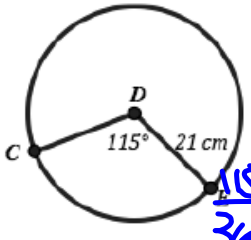
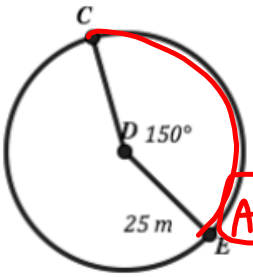


Find the measure of the length of each arc indicated below.

25.



26.