

SIMILARITY & RIGHT TRIANGLE TRIGONOMETRY - 6.10

6.10 Finding the Value of a Relationship



**Part 1: Pick a side**

Andrea and Bonita are resting under their favorite tree before taking a nature walk up a hill. Both girls have been studying trigonometry in school, and now it seems like they see right triangles everywhere. For example, Andrea notices the length of the shadow of the tree they are sitting under and wonders if they can calculate the height of the tree just by measuring the length of its shadow.

Bonita thinks they also need to know the measure of an angle, so she checks an app on her phone and finds that the angle of elevation of the sun at the current location and time of day is  $50^\circ$ . In the meantime, Andrea has paced off the length of the tree's shadow and finds that it is 40 feet long.

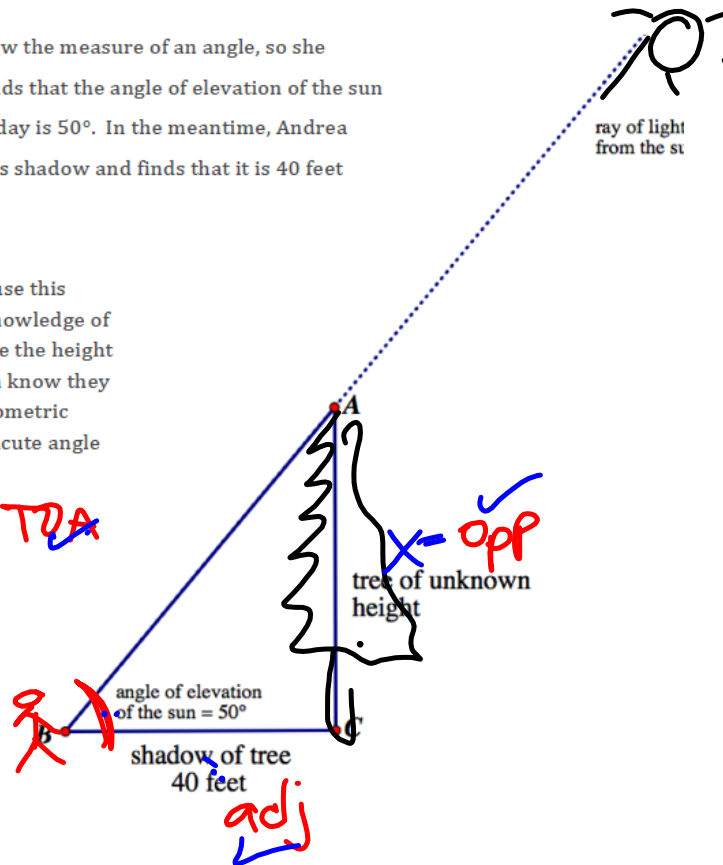
1. How might Andrea and Bonita use this information, along with their knowledge of trigonometric ratios, to calculate the height of the tree? (Andrea and Bonita know they can find the value of any trigonometric ratios they might need for any acute angle using a calculator.)

SOH CAH TOA

$$40(\tan 50) = \frac{x}{40} \cdot 40$$

$$x = 47.7 \text{ feet}$$

Mathematics Vision Project



✓ Part 2: What's your angle?

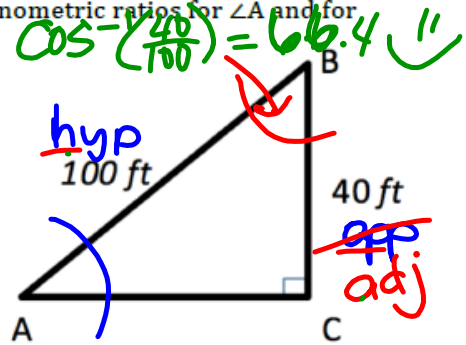
After their rest, Andrea and Bonita are going for a walk straight up the side of the hill. Andrea decided to stretch before heading up the hill while Bonita thought this would be a good time to get a head start. Once Bonita was 100 feet away from Andrea, she stopped to take a break and looked at her GPS device that told her that she had walked 100 feet and had already increased her elevation by 40 feet. With a bit of time to waste, Bonita wrote down the trigonometric ratios for  $\angle A$  and for  $\angle B$ .

SOH CAH TOA

2. Name the trigonometric ratios for  $\angle A$  and for  $\angle B$ .

$$\sin A = \frac{40}{100}$$

$$A = \sin^{-1}\left(\frac{40}{100}\right) = 23.6^\circ$$



When Andrea caught up, she said "What about the unknown angle measures? When I was at the bottom and looked up to see you, I was thinking about the "upward" angle measure from me to you. Based on your picture, this would be  $\angle A$ ."

Bonita wrote the trigonometric ratio  $\sin A = \frac{40}{100}$  and asked, "So, how do we find angle A?"

Together, the girls talked about how this was like thinking backwards: instead of knowing an angle and using their calculators to find a trigonometric ratio like they did while working on the height of the tree problem, they now know the trigonometric ratio and need to find an unknown angle value. Bonita notices the  $\sin^{-1}(\theta)$  button on her calculator and wonders if this might work like an "inverse trigonometric ratio" button, undoing the ratio to produce the angle. She decides to try it out, and produces the following output on her calculator:

$$\sin A = \frac{40}{100}$$

$$\sin^{-1}\left(\frac{40}{100}\right) = 23.578^\circ$$

$$\sin(23.578^\circ) = 0.4 = \frac{40}{100}$$

3. How might this output convince Bonita that her assumption about the calculator was correct?

4. Use the trigonometric ratio you found for  $\cos B$  to find the value of  $\angle B$ .

$$\cos^{-1}\left(\frac{40}{100}\right) = 66.4^\circ$$

inverse = inside angle

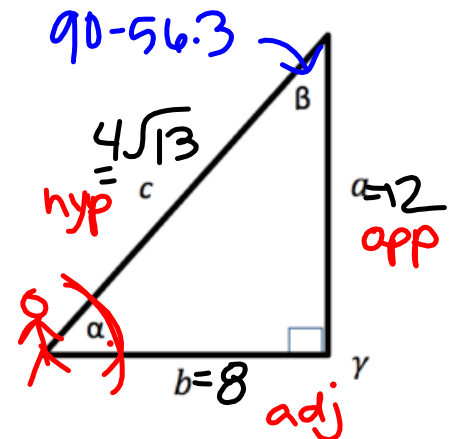
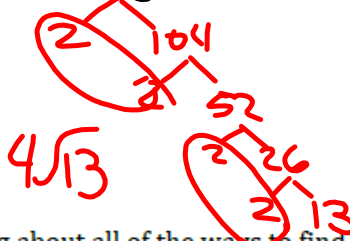
5. Find all unknown values for the following right triangle:

- a)  $\angle \alpha = 56.3^\circ$
- b)  $\angle \beta = 33.7^\circ$
- c)  $\angle \gamma = 90^\circ$
- d)  $a = 12\text{ m}$
- e)  $b = 8\text{ m}$
- f)  $c = 4\sqrt{13}\text{ m}$

$$8^2 + 12^2 = c^2$$

$$64 + 144 = c^2$$

$$\sqrt{208} = c$$



6. Bonita and Andrea started talking about all of the ways to find unknown values in right triangles and decided to make a list. What do you think should be on their list? Be specific and precise in your description. For example, 'trig ratios' is not specific enough. You may use the following sentence frame to assist with writing each item in your list:

When given \_\_\_\_\_, you can find \_\_\_\_\_ by \_\_\_\_\_.



$$A = \sin^{-1}\left(\frac{12}{4\sqrt{13}}\right) = 56.3^\circ$$

$$\cos^{-1}\left(\frac{8}{4\sqrt{13}}\right) = 56.3^\circ$$

$$\tan^{-1}\left(\frac{12}{8}\right) = 56.3^\circ$$

**Part 3: Angle of elevation and angle of depression**

During their hike, Andrea mentioned that she looked up to see Bonita. In mathematics, when you look straight ahead, we say your line of sight is a horizontal line. From the horizontal, if you look up, the angle from the horizontal to your line of sight is called the **angle of elevation**. Likewise, if you are looking down, the angle from the horizontal to your line of sight is called the **angle of depression**.

7. After looking at this description, Andrea mentioned that her angle of elevation to see Bonita was about  $23.5^\circ$ . They both agreed. Bonita then said her angle of depression to Andrea was about  $66.5^\circ$ . Andrea agreed that it was an angle of depression but said Bonita's angle of depression was also  $23.5^\circ$ . Who do you think is correct? Use drawings and words to justify your conclusion.
8. What conclusion can you make regarding the angle of depression and the angle of elevation? Why?

SECONDARY MATH II // MODULE 6  
SIMILARITY & RIGHT TRIANGLE TRIGONOMETRY - 6.10

6.10

READY, SET, GO!	Name	Period	Date
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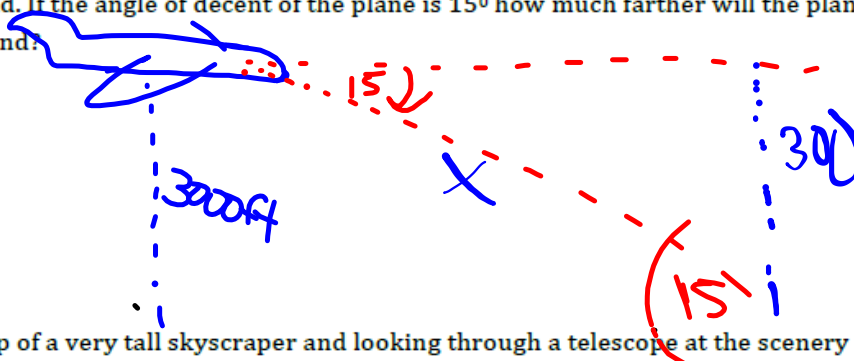
**READY**

Topic: Modeling real world problems with triangles.

For each story presented below *sketch a picture of the situation and label as much of the picture as possible.* **No need to answer the question or find the missing values, simply represent the situation with a sketch.**

- Jill put a ladder up against the house to try and reach a light that is out and needs to be changed. She knows the ladder is 10 feet long and the distance from the base of the house to the bottom of the ladder is 4 feet.

- Francis is a pilot of an airplane that if flying at an altitude of 3,000 feet when the plane begins its descent toward the ground. If the angle of decent of the plane is  $15^\circ$  how much farther will the plane fly before it is on the ground?



- Abby is standing at the top of a very tall skyscraper and looking through a telescope at the scenery all around her. The angle of decline on the telescope says  $35^\circ$  and Abby knows she is 30 floors up and each floor is 15 feet tall. How far from the base of the building is the object that Abby is looking at?

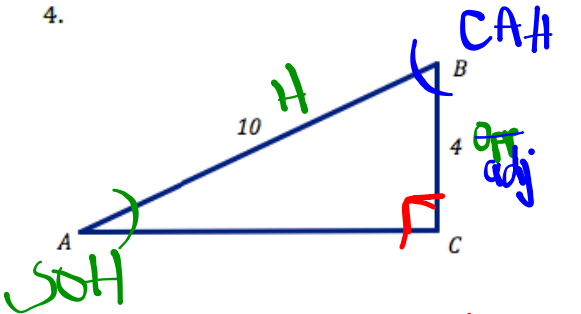
SET

Topic: Solving triangles using Trigonometric Ratios

In each triangle find the missing angles and sides.

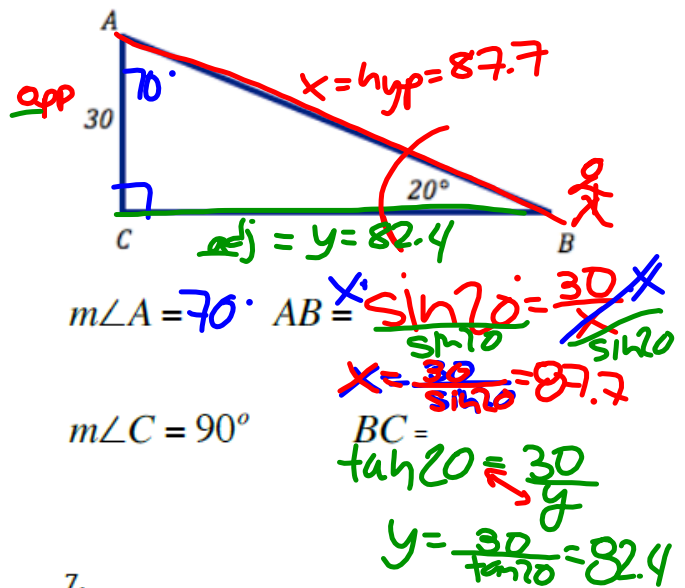
SOH CAH TOA

4.



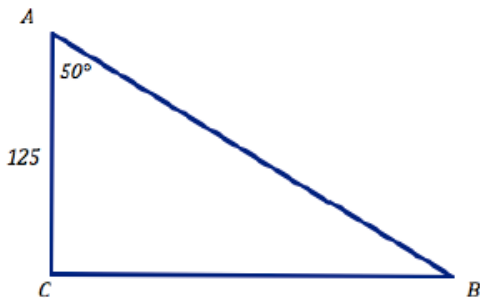
$m\angle A = \sin^{-1}\left(\frac{4}{10}\right) = 23.6^\circ$   
 $m\angle B = \cos^{-1}\left(\frac{4}{10}\right) = 66.4^\circ$   
 $m\angle C = 90^\circ$  ✓  $AC = 4$   
 $4^2 + x^2 = 10^2 \dots$

5.



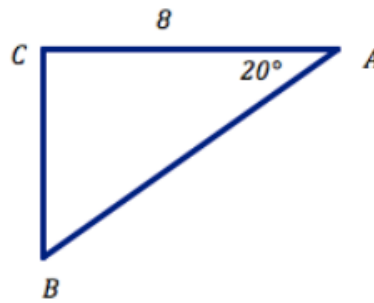
$m\angle A = 70^\circ$   
 $AB = \frac{30}{\sin 70^\circ} = 87.7$   
 $m\angle C = 90^\circ$   
 $BC = \frac{30}{\tan 70^\circ} = 82.4$

6.



$AB =$   $m\angle B =$   
 $m\angle C = 90^\circ$   $BC =$

7.



$AB =$   $m\angle B =$   
 $m\angle C = 90^\circ$   $BC =$

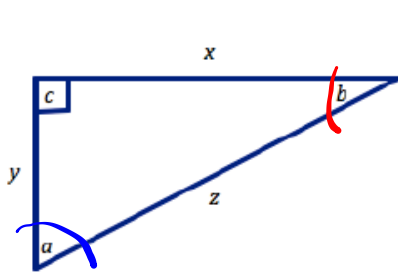
GO

Topic: Trigonometric ratios

SOH CAH TOA

Use the given right triangle to identify the trigonometric ratios. And angles were possible.

8.



$$\sin(a) = \frac{x}{z}$$

$$\cos(a) = \frac{y}{z}$$

$$\tan(a) = \frac{x}{y}$$

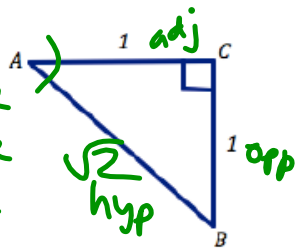
$$\sin(b) = \frac{y}{z}$$

$$\cos(b) = \frac{x}{z}$$

$$\tan(b) = \frac{y}{x}$$

9.

$$\begin{aligned} 1^2 + 1^2 &= c^2 \\ 1 + 1 &= c^2 \\ 2 &= c^2 \\ \sqrt{2} &= c \end{aligned}$$



$$\sin^{-1}\left(\frac{1}{\sqrt{2}}\right) = 45^\circ$$

$$\sin(A) = \frac{1}{\sqrt{2}}$$

$$\cos(A) = \frac{1}{\sqrt{2}}$$

$$\tan(A) = \frac{1}{1} = 1$$

$$\sin(B) =$$

$$\cos(B) =$$

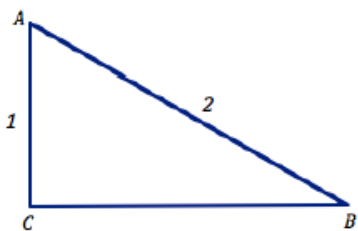
$$\tan(B) =$$

$$m\angle A = 45^\circ$$

$$m\angle B =$$

inverse of ANY of these fractions

10.



$$\sin(A) =$$

$$\cos(A) =$$

$$\tan(A) =$$

$$\sin(B) =$$

$$\cos(B) =$$

$$\tan(B) =$$

$$m\angle A =$$

$$m\angle B =$$

$$m\angle C = 90^\circ$$