4.2 Bike Lovers

A Solidify Understanding Task

Michelle and Rashid love going on long bike rides.

Every Saturday, they have a particular route they bike together that takes four hours. Below is a piecewise function that estimates the distance they travel for each hour of their bike ride.

$$f(x) = \begin{cases} 16x, & 0 \le x \le 1 \\ 10(x-1)+16, & 1 < x \le 2 \\ 14(x-2)+26, & 2 < x \le 3 \\ 12(x-3)+40 & 3 < x \le 4 \end{cases}$$

1. What part of the bike ride are they going the fastest? Slowest?

   $16x$ ✓ $\quad$ $10x$

2. What is the domain of this function?

   $[0, 4]$ $\quad$ $0 \le x \le 4$

3. Find f(2). Explain what this means in terms of the context.

   $f(2) = 10(2-1)+16$
   $10(1)+16 = 26$ miles
   after 2 hours, the distance is 26 miles.

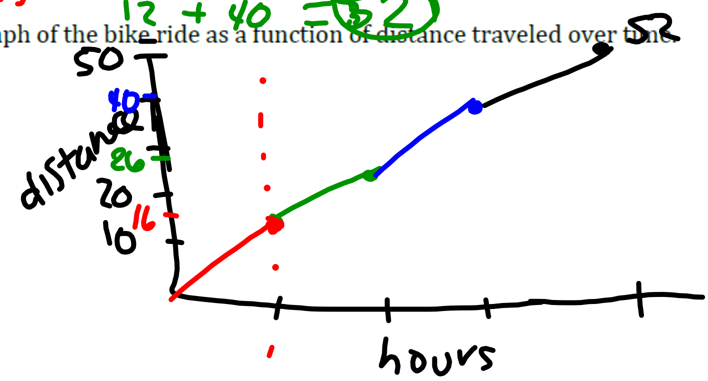
4. How far have they traveled at 3 hours? Write the answer using function notation.

   $f(3) = 14(3-2)+26$
   $14+26 = 40$

5. What is the total distance they travel on this bike ride?

   $f(4) = 12(x-3)+40$
   $12+40 = 52$

6. Sketch a graph of the bike ride as a function of distance traveled over time.



Rashid also has a route he likes to do on his own and has the following continuous piecewise function to represent the average distance he travels in minutes:

$\frac{1}{4}$ $\frac{1}{5}$ $\frac{2}{7}$ $\frac{1}{8}$

.25 .2 .29 .125

$$g(x) = \begin{cases} \frac{1}{4}(x) & 0 \le x \le 20 \\ \frac{1}{5}(x-20)+5 & 20 < x \le 50 \\ \frac{2}{7}(x-50)+11 & 50 < x \le 92 \\ \frac{1}{8}(x-a)+b & 92 < x \le 100 \end{cases}$$

92 23

$\frac{2}{7}(92-50)+11=$

$\frac{2}{\cancel{7}}\cancel{42}\,6$

$12+11=23$

7. What is the domain for this function? What does the domain tell us?

$0 \le x \le 100$ or $[0, 100]$

8. What is the average rate of change during the interval [20, 50]?

$\frac{1}{5} = m$

9. Over which time interval is the greatest average rate of change?

$\frac{2}{7} > \frac{1}{4} > \frac{1}{5} > \frac{1}{8}$

fastest slowest

10. Find the value of each, then complete each sentence frame:

$\frac{1}{5}(30-20)+5$

$2+5$

a. g(30) = 7. This means... after 30 times, the distance is 7 miles.

b. g(64)= __________. This means...

c. g(10) = _________. When finding output values for given input values in a piecewise function, you must ...

11. Complete the last equation by finding values for a and b.

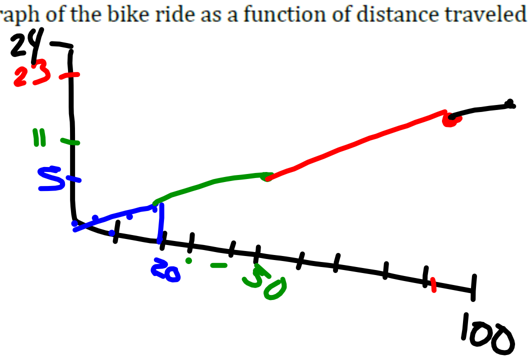
$a = 92 \quad b = 23$

12. Sketch a graph of the bike ride as a function of distance traveled as a function of time.

$\frac{1}{8}(100-92)+23$

$\frac{1}{\cancel{8}}\cancel{8}\,1 + 23$

24



h(x) represents distance traveled in km, to answer the following questions.

$$h(x) = \begin{cases} \frac{1}{4}x^2 & 0 \le x \le 10 \\ \frac{1}{2}(x - 10) + c & 10 < x \le 20 \\ 2(x - 20) + 30 & 20 < x \le 30 \end{cases}$$

13. Find the value of c.

14. Sketch the graph (label axes).

15. What is the domain of h(x)?

16. What is the range of h(x)?

17. Which five minute interval of time has the greatest average rate of change?

    a. [0, 5] b. [5, 10] c. [10, 15] d. [25, 30]

    What is the average rate of change over this interval?

18. Find h(8).

19. Find h(15).

READY, SET, GO! Name Period Date

READY

Topic: Solving absolute value equations.

Solve for x. (You will have two answers.)

1. $|x| = 7$ 7, −7

2. $|x - 6| = 3$

3. $|w + 4| = 11$

4. $-9|m| = -63$ $|m| = \pm 7$ $\boxed{7, -7}$

5. $|3d| = 15$

6. $|3x - 5| = \pm 11$ $3x-5=11$, $+5 \; +5$, $3x = 16$, $\boxed{x = \frac{16}{3}}$ | $3x - 5 = -11$, $+5 \; +5$, $\frac{3x}{3} = \frac{-6}{3}$, $\boxed{x = -2}$

7. $-|m + 3| = -13$

8. $|-4m| = 64$

9. $2|x + 1| - 7 = -3$

10. $5|c + 3| - 1 = 9$ $+1 \; +1$ $\frac{5|c+3|}{5} = \frac{10}{5}$ $|c+3| = \pm 2$ $-3 \; -3$ $\boxed{-1, -5}$

11. $-2|2p - 3| - 1 = -11$

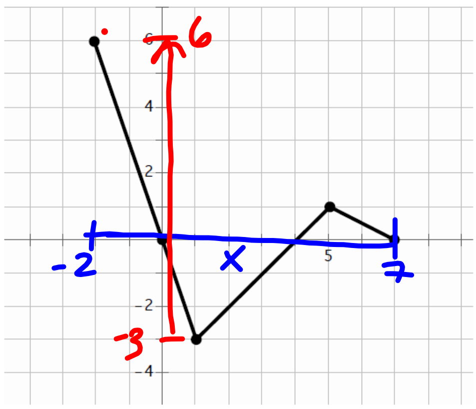
12. Explain why the equation $|m| = -3$ has no solution.

distance is positive

SET

Topic: Reading the domain and range from a graph

State the domain and range of the piece-wise functions in the graph. Use interval notation.



a. Domain:

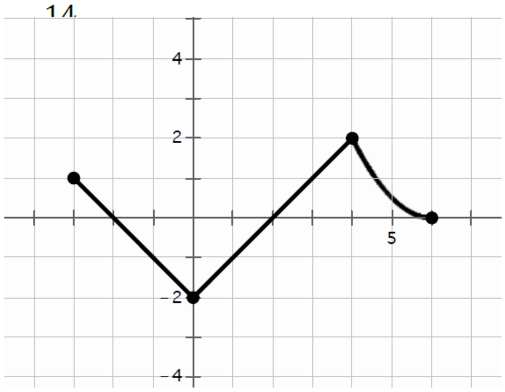
$-2 \leq x \leq 7$

$[-2, 7]$

b. Range:

$-3 \leq y \leq 6$

14.



a. Domain:

b. Range:

For each of the graphs below write the interval that defines each piece of the graph. Then write the domain of the entire piece-wise function.

Example: (Look at the graph in #14. Moving left to right. Piece-wise functions use set notation.)

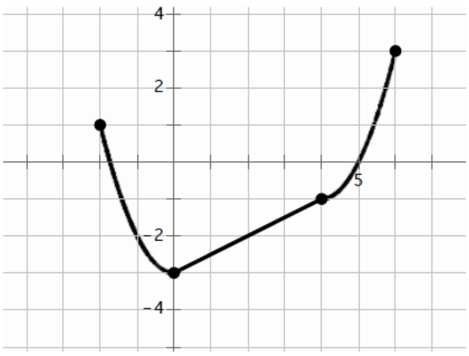
Interval 1 $-3 \leq x < 0$

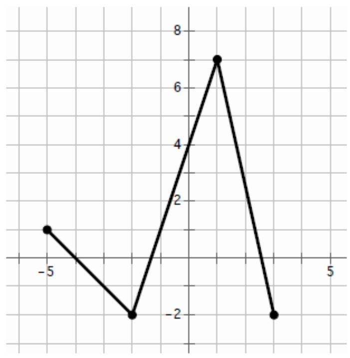
Interval 2 $0 \leq x < 4$

Interval 3 $4 \leq x \leq 6$

Domain: $[-3,6]$ (We can use interval notation on the domain, if it's continuous.)

Pay attention to your inequality symbols! You do not want the pieces of your graph to overlap. Do you know why? it won't pass the verticle line test, so not a function.



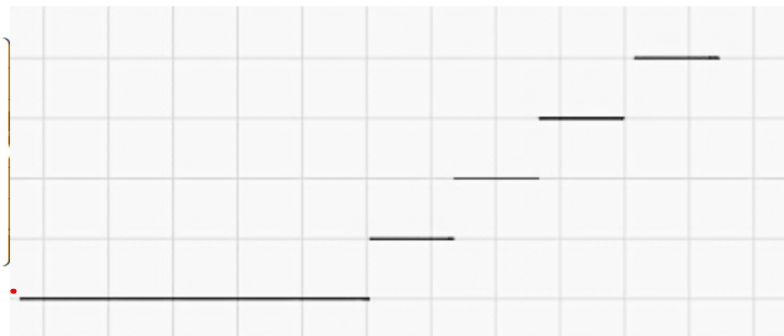


15. a. Interval 1 ______________
    b. Interval 2 ______________
    c. Interval 3 ______________
    d. Domain: ______________

16. a. Interval 1 ______________
    b. Interval 2 ______________
    c. Interval 3 ______________
    d. Domain: ______________

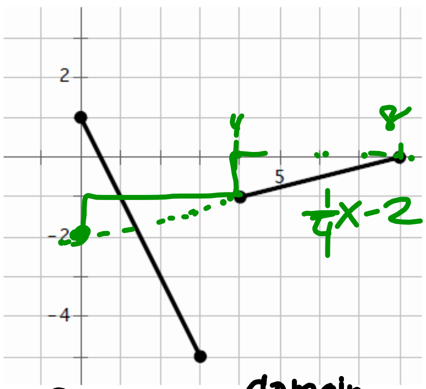
17. So far you've only seen continuous piece-wise defined functions, but piece-wise functions can also be non-continuous. In fact, you've had some real life experience with one kind of non-continuous piece-wise function. The graph below represents how some teachers calculate grades. Finish filling in the piece-wise equation. Then label the graph with the corresponding values.

$$f(x)=\begin{cases} A, & 90 \le x \quad 100 \\ B, & 80 \le x < 90 \\ C, & 70 \le x < 80 \\ D, & 60 \le x < 70 \\ F, & 0 \le x < 60 \end{cases}$$



Write the piece-wise equations for the given graphs.

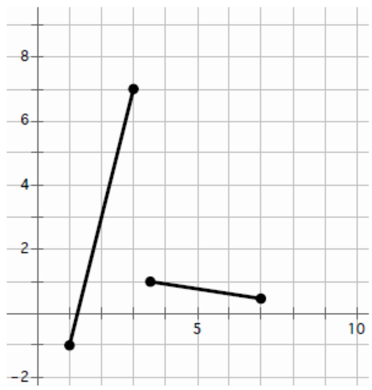
18.



domain

$$fx\begin{cases} & , \\ \frac{1}{4}x-2 & , 4 \le x \le 8 \end{cases}$$

19.



GO

Topic: Transformations on quadratic equations

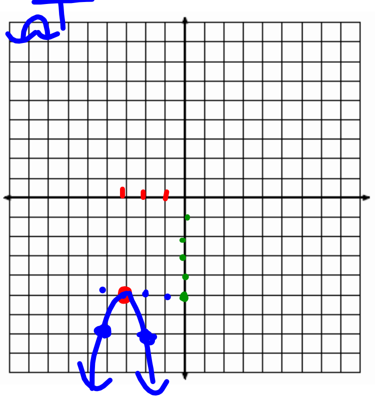
$y = a(x-h)^2 + k$ ← up/down

stretch   opp side

Beginning with the parent function $f(x) = x^2$, write the equation of the new function $g(x)$ that is a transformation of $f(x)$ as described. Then graph it.

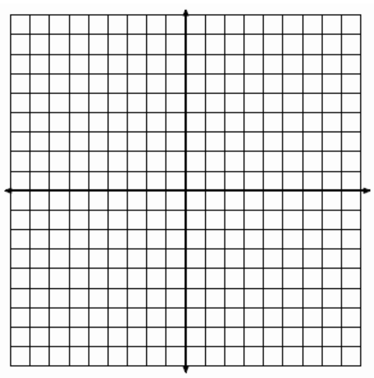
20. Shift $f(x)$ left 3 units, stretch vertically by 2, reflect $f(x)$ vertically, and shift down 5 units.

$g(x) = -2(x+3)^2 - 5$



21. Shift $f(x)$ right 1, stretch vertically by 3, and shift up 4 units.

$g(x) =$ ________________



22. Shift $f(x)$ up 3 units, left 6, reflect vertically, and stretch by ½

$g(x) =$ ________________