3.7 Perfecting My Quads

A Practice Understanding Task

Carlos and Clarita, Tia and Tehani, and their best friend Zac are all discussing their favorite methods for solving quadratic equations of the form $ax^2 + bx + c = 0$. Each student thinks about the related quadratic function $y = ax^2 + bx + c$ as part of his or her strategy.

Complete the first table using each strategy

Carlos: "I like to make a table of values for *x* and find the solutions by inspecting the table."

Zac: "I like to graph the related quadratic function and use my graph to find the solutions."

Clarita: "I like to write the equation in factored form, and then use the factors to find the solutions."

Tia: "I like to treat it like a quadratic function that I put in vertex form by completing the square. I can then use a square root to undo the squared expression."

Tehani: "I also like to use the quadratic formula to find the solutions."

Lesson 3.7: Complete the first table using each strategy

Demonstrate how each student might solve each of the following quadratic equations.

Solve:

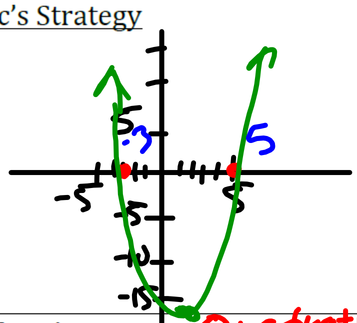
a b c

$1x^2 - 2x - 15 = 0$

$(-4)^2 - 2(-4) - 15$

-5	$(-5)^2 - 2(-5) - 15 = 20$
-4	9
-3	0 ←
-2	-7
-1	-12
0	-15
1	-16 vertex
2	-15
3	-12
4	-7
5	0 ←

Zac's Strategy



Clarita's Strategy factoring

$(x+3)(x-5)$

$x = -3, 5$

-15, 3, -5, -2

Tia's Strategy vertex

$(x^2 - 2x + 1) - 15 - 1$

$\square 1$

0

$\sqrt{(x-1)^2} - 16 = 0$

$+16 \quad \sqrt{16}$

$x - 1 = \pm 4$

$+1 \quad +1$

$x = 1 + 4, \; 1 - 4$

$x = 5, -3$ ✓

Tehani's Strategy Quadratic

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$x = \dfrac{-(2) \pm \sqrt{(-2)^2 - 4(1)(-15)}}{2(1)}$

$\sqrt{4+60}$

$x = \dfrac{2 \pm \sqrt{64}}{2}$

$x = \dfrac{2 \pm 8}{2}$

$x = \dfrac{10}{2}, -\dfrac{6}{2}$

$\boxed{x = 5, -3}$ ✓

Solve:

$2x^2 + 3x + 1 = 0$

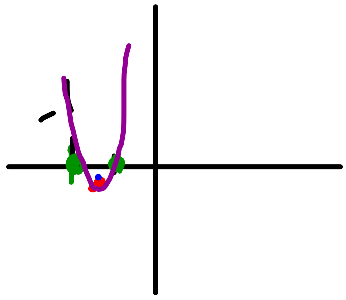
a b c

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Carlos' Strategy

-3	10
-2	3
-1	0
0	1
1	6
2	15
3	28

Zac's Strategy



Clarita's Strategy factored

$0 = (2x+1)(1x+1)$

$0 = 2x+1 \quad x+1=0$

$-\frac{1}{2} = x \quad x = -1$

2 : 1

$\frac{1}{2}$ $\frac{1}{2}$ 3

check

$2x^2 + 2x + x + 1$

$2x^2 + 3x + 1$ ✓

Tia's Strategy vertex

$(2x^2 + 3x + \frac{18}{16}) + 1 - \frac{18}{16}$

$2(x^2 + \frac{3}{2}x + \frac{9}{16})$ $-\frac{2}{16}$

half, squared

$2(x + \frac{3}{4})^2 - \frac{1}{8} = 0$ $+\frac{1}{8}$ $/2$

$\sqrt{(x + \frac{3}{4})^2} = \sqrt{\frac{1}{16}}$

$x + \frac{3}{4} = \pm\frac{1}{4}$

$-\frac{3}{4} \quad -\frac{3}{4}$

$x = -\frac{3}{4} + \frac{1}{4}, \quad -\frac{3}{4} - \frac{1}{4}$

$x = -\frac{2}{4} \sim -\frac{1}{2} \quad -\frac{4}{4} \sim -1$

Tehani's Strategy Quad

$x = \dfrac{-3 \pm \sqrt{3^2 - 4(2)(1)}}{2(2)}$

$9 - 8$ $\sqrt{1}$

$x = \dfrac{-3 \pm \sqrt{1}}{4}$

$x = \dfrac{-3+1}{4}, \dfrac{-3-1}{4}$

$x = -\frac{2}{4}, -\frac{4}{4}$

$x = -\frac{1}{2}, -1$

Solve:

a b c

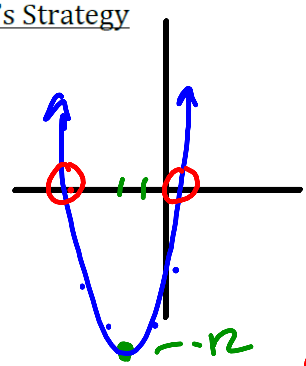
$1x^2 + 4x - 8 = 0$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Carlos' Strategy

-6	
2	

Zac's Strategy



Clarita's Strategy

-8

~~1~~ 8

~~2~~ -4

CAN'T

Tia's Strategy

vertex

$(x^2 + 4x + 4) - 8 - 4$

half 2 squared

$(x+2)^2 - 12 = 0$

$+12 \quad +12$

$\sqrt{(x+2)^2} = \sqrt{12}$

3, 4, (2)

$x + 2 = \pm 2\sqrt{3}$

$-2 \quad -2$

$x = -2 + 2\sqrt{3}, -2 - 2\sqrt{3}$

Tehani's Strategy

Quad

$x = \frac{-4 \pm \sqrt{4^2 - 4(1)(-8)}}{2(1)}$

16+32

$\frac{-4 \pm \sqrt{48}}{2}$

8, 6, 4, 2, 2, 2, 3

$\frac{-4}{2} \pm \frac{4\sqrt{3}}{2}$

$\boxed{x = -2 \pm 2\sqrt{3}}$

Describe why each strategy works.

As the students continue to try out their strategies, they notice that sometimes one strategy works better than another. Explain how you would decide when to use each strategy.

The solution is where the 2 lines intersect.

Here is an extra challenge. How might each student solve the following system of equations?

Solve the system:

where they are equal to each other

$y_1 = x^2 - 4x + 1$

$y_2 = x - 3$

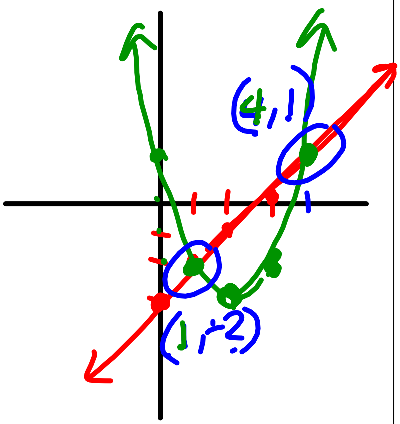
$x^2 - 4x + 1 = x - 3$

$\quad -x + 3 \quad\quad -x + 3$

$x^2 - 5x + 4 = 0$

Carlos' Strategy

Zac's Strategy



Clarita's Strategy Factor

$(x-1)(x-4)=0$ -1, -4, 4, -5

$1, 4$

$y = 1 - 3 \qquad y = 4 - 3$

$y = -2 \qquad y = 1$

$(1,-2) \qquad (4,1)$ ✓

Tia's Strategy vertex

$(x^2 - 4x + 4) + 1 - 4$

half

$(x-2)^2 - 3$

Tehani's Strategy

SECONDARY MATH II // MODULE 3
SOLVING QUADRATICS & OTHER EQUATIONS – 3.7

3.7

READY, SET, GO! Name Period Date

READY

Topic: Symmetry and Distance

The given functions provide the connection between possible areas, *A(x)*, that can be created by a rectangle for a given side length, *x*, and a set amount of perimeter. You could think of it as the different amounts of area you can close in with a given amount of fencing as long as you always create a rectangular enclosure.

1. $A(x) = x(10 - x)$ $= -x^2 + 10x$

Find the following:

a. $A(3) =$ b. $A(4) =$

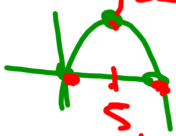
$3(10-3)$
$3(7)$
$=21$

c. $A(6) =$ d. $A(x) = 0$

$x(10-x) = 0$
$x = 0, 10$
$(x+0)(10-x)$

e. When is A(x) at its maximum? Explain or show how you know.

5, 25



5,

$\boxed{x=5}$

$5(10-5)$
$A = 5(5) = 25$

2. $A(x) = x(50 - x)$

Find the following:

a. $A(10) =$ b. $A(20) =$

c. $A(30) =$ d. $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

3. $A(x) = x(75 - x)$

Find the following:

a. $A(20) =$

b. $A(35) =$

c. $A(40) =$

d. $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

4. $A(x) = x(48 - x)$

Find the following:

a. $A(10) =$

b. $A(20) =$

c. $A(28) =$

d. $A(x) = 0$

e. When is A(x) at its maximum? Explain or show how you know.

SET

Topic: Solve Quadratic Equations Efficiently

For each of the given quadratic equations find the solutions using an efficient method. State the method you are using as well as the solutions. You must use at least three different methods.

5. $x^2 + 17x + 60 = 0$

6. $x^2 + 16x + 39 = 0$

7. $x^2 + 7x - 5 = 0$

$x = \frac{-7 \pm \sqrt{7^2 - 4(1)(-5)}}{2}$

49+20

$\boxed{x = \frac{-7 \pm \sqrt{69}}{2}}$

8. $3x^2 + 14x - 5 = 0$

$0 = (3x - 1)(x + 5)$

-15, 14, 15, -1

$\frac{-1}{3}$, $\frac{15}{3}$, $\frac{5}{1}$

$\boxed{x = +\frac{1}{3}, -5}$

9. $x^2 - 12x = -8$

10. $x^2 + 6x = 7$

Summarize the process for solving a quadratic by the indicated strategy. Give examples along with written explanation, also indicate when it is best to use this strategy.

11. Completing the Square

If b is even & a = 1 then this way is easier.

12. Factoring

if factors of (a·c) add to make B

13. Quadratic Formula

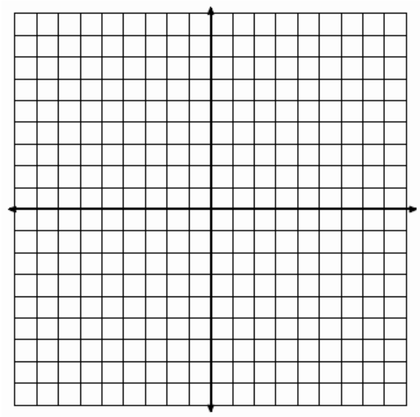
If its not any of the above use quad.

GO

Topic: Graphing Quadratics and finding essential features of the graph. Solving systems of equations.

Graph the quadratic function and supply the desired information about the graph.

14. $f(x) = x^2 + 8x + 13$



a. Line of symmetry:

b. x-intercepts:

c. y-intercept:

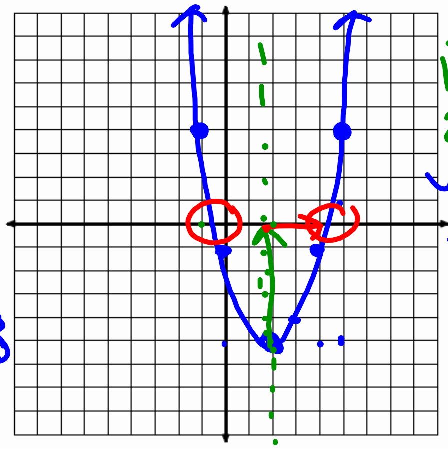
d. vertex:

$-1 \quad -4$

4

15. $f(x) = x^2 - 4x - 1$

$0 = (x-2)^2 - 5 \rightarrow (2,-5)$



$1^2 = 1$

$2^2 = 4$

$\sqrt{x^2} = \sqrt{5}$

$x = \sqrt{5}$

a. Line of symmetry: $x = 2$

b. x-intercepts: $2 \pm \sqrt{5}$

$x = \dfrac{4 \pm \sqrt{(-4)^2 - 4(1)(-1)}}{2}$

$16+4$

$\dfrac{4 \pm \sqrt{20}}{2}$ $\quad 20 < \begin{matrix} 4 & 2, 2 \\ 5 \end{matrix}$

$\dfrac{4}{2} \pm \dfrac{2\sqrt{5}}{2}$

$\boxed{2 \pm \sqrt{5}}$

c. y-intercept: $(0,-1)$

d. vertex: $(2,-5)$

Solve each system of equations using an algebraic method and check your work!

16. Elimination

$$\begin{cases} 3y + 5y = 15 \\ 3x + 2y = 6 \end{cases}$$

$7y = 9$

$y = 9/7$ sub back in

$3x + 5\left(\frac{9}{7}\right) = 15$

$3x + \frac{45}{7} = \frac{7(15)}{7} - \frac{45}{7}$

$3x = \frac{105-45}{7} = \frac{60}{7} \cdot \frac{1}{3}$

$x = \frac{60}{21} \div 3$

$x = \frac{20}{7}$

$\boxed{\left(\frac{20}{7}, \frac{9}{7}\right)}$

17.

$$\begin{cases} y = -7x + 12 \\ y = 5x - 36 \end{cases}$$

18. Substitution

$$\begin{cases} y = 2x + 12 \\ y = 10x - x^2 \end{cases}$$

$2x + 12 = 10x - x^2 - 12$

$-2x \quad -12 \qquad -2x$

$0 = -x^2 + 8x - 12$

$-(x^2 - 8x + 12)$

$-(x-2)(x-6)$

$x = 2, 6$

$y = 2(2) + 12$

$y = 16$

$y = 2(6) + 12$

$y = 24$

$(2, 16) \quad (6, 24)$

19.

$$\begin{cases} y = 24x - x^2 \\ y = 8x + 48 \end{cases}$$

$24x - x^2 = 8x + 48$

$-8x \quad -48$

$-x^2 + 16x - 48 = 0$

$-(x^2 - 16x + 48) = 0$

$-(x-4)(x-12)$

$x = 4, 12$

$y = 8(4) + 48$

$32 + 48$

$y = 80$

$(4, 80)$

$y = 8(12) + 48$

$96 + 48$

$y = 144$

$(12, 144)$