3.5 Throwing an Interception

A Develop Understanding Task

Start lesson 3.5

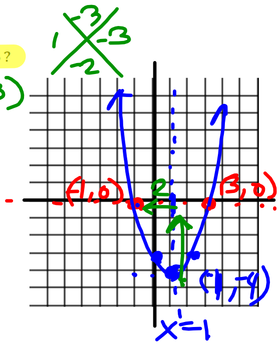
The x-intercept(s) of the graph of a function $f(x)$ are often very important because they are the solution to the equation $f(x)=0$. In past tasks, we learned how to find the x-intercepts of the function by factoring, which works great for some functions, but not for others. In this task we are going to work on a process to find the x-intercepts of any quadratic function that has them. We'll start by thinking about what we already know about a few specific quadratic functions and then use what we know to generalize to all quadratic functions with x-intercepts.

1. What can you say about the graph of the function $f(x)=x^2-2x-3$?

   a. Graph the function

   $0=(x+1)(x-3)$

   $x=-1, 3$

   b. What is the equation of the line of symmetry?

   $x=1$

   c. What is the vertex of the function?

   $(1,-4)$

2. Now let's think specifically about the x-intercepts.

   a. What are the x-intercepts of $f(x)=x^2-2x-3$?

   $(-1,0)$ $(3,0)$

   b. How far are the x-intercepts from the line of symmetry?

   2

   c. If you knew the line of symmetry was the line $x=h$, and you know how far the x-intercepts are from the line of symmetry, how would you find the actual x-intercepts?

   $1-2$, $1+2$

   -1 , 3

   d. How far above the vertex are the x-intercepts?

   4 up.

   e. What is the value of $f(x)$ at the x-intercepts?

   $y=?$

   $y=0$ always at the x intercepts.

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3. So, let's think about another function: $f(x)=x^2-6x+4$

   4
   CAN'T
   -6

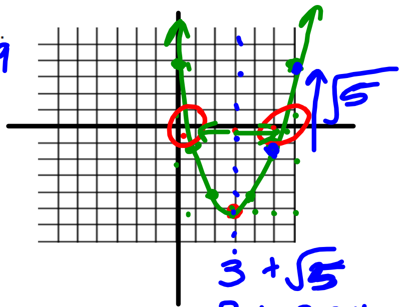
   a. Graph the function by putting the equation into vertex form.

   $(x^2-6x+9)+4-9$

   $\boxed{(x-3)^2-5}$

   +4 -9
   -5
   = 9

   $(x-3)^2-5$

   b. What is the vertex of the function? vertex $(3,-5)$

   c. What is the equation of the line of symmetry?

   $x=3$

   d. What do you estimate the x-intercepts of the function to be?

   ~ .8 ~5.2 $3\pm\sim2.5$

   e. What do you estimate d to be?

   $\sqrt{9-4}=\sqrt{5}$

   f. What is the value of $f(x)$ at the x-intercepts?

   0

   g. Using the vertex form of the equation and your answer to part "f" above, write an equation and solve it to find the exact values of the x intercepts.

   $(x-3)^2-5=0$
   $+5 \quad +5$
   $\sqrt{(x-3)^2}=\sqrt{5}$

   $x-3=\pm\sqrt{5}$
   $+3 \quad +3$
   $\boxed{x=3\pm\sqrt{5}}$

   h. What is the exact value of d?

   $\sqrt{5}=\sim2.24$

   i. Use a calculator to find approximations for the x-intercepts. How do they compare with your estimates?

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4. What about a function with a vertical stretch? Can we find exact values for the x-intercepts the same way? Let's try it with: $f(x) = 2x^2 - 8x + 5$.

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a. Graph the function by putting the equation into vertex form.

$(2x^2 - 8x + 8) + 5 - 8$

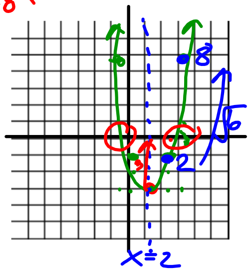
$2(x - 2)^2 - 3$

$2(\;\;)^2 + 5 - 8$

-3

$2(x-2)^2 - 3$

$(2, -3)$



b. What is the vertex of the function?

2

c. What is the equation of the line of symmetry?

$x = 2$

d. What do you estimate the x-intercepts of the function to be?

$2 \pm \sqrt{6}$ too big 2.44

$2 \pm \sqrt{3}$ ~1.73 too big

e. What do you estimate d to be?

$\frac{\sqrt{6}}{2} \sim 1.22$ $\sim 2 \pm \frac{\sqrt{6}}{2} \sim 1.22$

f. What is the value of $f(x)$ at the x-intercepts?

$y = 0$

g. Using the vertex form of the equation and your answer to "f" above, write an equation and solve it to find the exact values of the x-intercepts.

$2(x-2)^2 - 3 = 0$

$+3 \quad +3$

$\frac{2(x-2)^2}{2} = \frac{3}{2}$

$x - 2 = \pm\sqrt{\frac{3}{2}}$

$+2$

$x = 2 \pm \sqrt{\frac{3}{2}}$

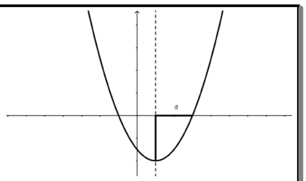
2 ± 1.22

h. What is the exact value of d?

$\boxed{\frac{\sqrt{2}\sqrt{3}}{2}}$ $\frac{\sqrt{2}\sqrt{3}}{\sqrt{2}\sqrt{2}}$ $\sqrt{\frac{3}{2}} = \frac{\sqrt{6}}{2}$ $\boxed{\frac{\sqrt{2}\sqrt{3}}{2}}$

i. Compare your solution to your estimate of the roots. How did you do?

5. Finally, let's try to generalize this process by using: $f(x) = ax^2 + bx + c$ to represent any quadratic function that has x-intercepts. Here's a possible graph of $f(x)$.



a. Start the process the usual way by putting the equation into vertex form. It's a little tricky, but just do the same thing with a, b, and c as what you did in the last problem with the numbers.

b. What is the vertex of the parabola?

c. What is the line of symmetry of the parabola?

d. Write and solve the equation for the x-intercepts just as you did previously.

6. How could you use the solutions you just found to tell what the x-intercepts are for the function $f(x) = x^2 - 3x - 1$?

Quadratic formula using complete the square

$ax^2 + bx + c$

$a\left(\square \ \frac{b}{2a}\right) + c - \left(\frac{b}{2a}\right)^2 a$

$\frac{b}{2a}$ $\left(\frac{b}{2a}\right)^2$

$a\left(x + \frac{b}{2a}\right)^2 + c - \frac{b^2}{(2a)^2} \cdot a$

$c - \frac{b^2}{4a^{\cancel{2}}} \cdot \cancel{a}$

idea

$\frac{3}{3}(2) - \frac{2}{3}$

$\frac{4a}{4a} \cdot \frac{c}{1} - \frac{b^2}{4a}$ get common denom

$a\left(x+\frac{b}{2a}\right)^2 + \frac{4ac - b^2}{4a} = 0$

$a\left(x+\frac{b}{2a}\right)^2 = -\frac{(4ac-b^2)}{4a}$

$-3+7$

$7-3$

$\sqrt{\frac{\cancel{a}}{\cancel{a}}\left(x+\frac{b}{2a}\right)^2} = \sqrt{\frac{-4ac + b^2}{4a^2}} \cdot \frac{1}{a}$

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{\sqrt{4a^2}}$

$x + \frac{b}{2a} = \pm\frac{\sqrt{b^2-4ac}}{2a}$

$x = -\frac{b}{2a} \pm \frac{\sqrt{b^2-4ac}}{2a}$

axis of symmetry distance

Quadratic formula $\boxed{x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}}$

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7. You may have found the algebra for writing the general quadratic function $f(x) = ax^2 + bx + c$ in vertex form a bit difficult. Here is another way we can work with the general quadratic function leading to the same results you should have arrived at in 5c

   a. Since the two *x*-intercepts are *d* units from the line of symmetry *x* = *h*, if the quadratic crosses the *x*-axis its *x*-intercepts are at (*h* – *d*, 0) and (*h* + *d*, 0). We can always write the factored form of a quadratic if we know its *x*-intercepts. The factored form will look like $f(x) = a(x-p)(x-q)$ where *p* and *q* are the two *x*-intercepts. So, using this information, write the factored form of the general quadratic $f(x) = ax^2 + bx + c$ using the fact that its *x*-intercepts are at *h*-*d* and *h*+*d*.

   b. Multiply out the factored form (you will be multiplying two **trinomial** expressions together) to get the quadratic in standard form. Simplify your result as much as possible by combining like terms.

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SECONDARY MATH II // MODULE 3
SOLVING QUADRATICS & OTHER EQUATIONS – 3.5

Start homework

3.5

READY, SET, GO! Name Period Date

READY

Topic: Converting measurement of area, area and perimeter.

While working with areas is sometimes essential to convert between units of measure, for example changing from square yards to square feet and so forth. Convert the areas below to the desired measure. (Hint: area is two dimensional, for example 1 yd² = 9 ft² because 3 ft along each side of a square yard equals 9 square feet.)

5280^2 ft= 1 mi²

1. 7 yd² = ? ft²

$7\cancel{yd^2} * \frac{9\,ft^2}{\cancel{yd^2}} = 63ft$

2. 5 ft² = ? in²

3. 1 mile² = ? ft²

4. 100 m² = ? cm²

$100\,\cancel{m^2} \times \frac{10000\,cm^2}{\cancel{1m^2}}$

$1m^2 = 100cm^2$

$1m^2 = 10000\,cm$

$1\,000\,000\,cm^2$

5. 300 ft² = ? yd²

6. 96 in² = ? ft²

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Solve the quadratic equations below.

9. $x^2 + 7x - 170 = 0$

10. $x^2 + 15x - 16 = 0$

11. $x^2 + 2x - 35 = 0$

12. $x^2 + 10x - 80 = 0$

GO

Topic: Factoring Expressions

Write each of the expressions below in factored form.

13. $x^2 - x - 132$

14. $x^2 - 5x - 36$

15. $x^2 + 5x + 6$

16. $x^2 + 13x + 42$

17. $x^2 + x - 56$

18. $x^2 - x$

19. $x^2 - 8x + 12$

20. $x^2 - 10x + 25$

21. $x^2 + 5x$

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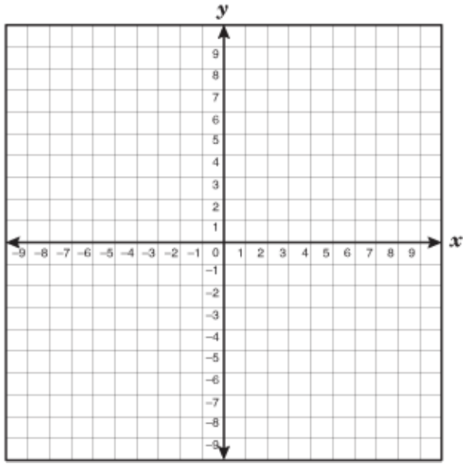
SET

Topic: Transformations and parabolas, symmetry and parabolas

7a. Graph each of the quadratic functions.

$$f(x) = x^2$$
$$g(x) = x^2 - 9$$
$$h(x) = (x + 2)^2 - 9$$



b. How do the functions compare to each other?

c. How do *g(x)* and *h(x)* compare to *f(x)*?

d. Look back at the functions above and identify the x-intercepts of *g(x)*. What are they?

e. What are the coordinates of the points corresponding to the x-intercepts in *g(x)* in each of the other functions? How do these coordinates compare to one another?

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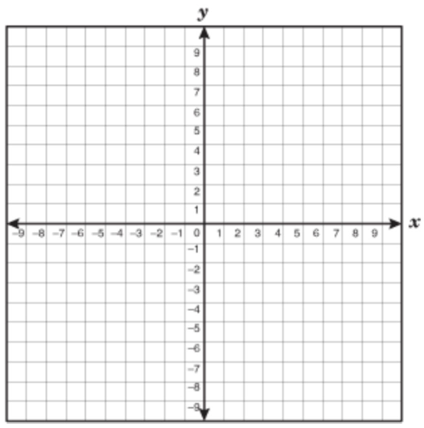
8a. Graph each of the quadratic functions.

$$f(x) = x^2$$
$$g(x) = x^2 - 4$$
$$h(x) = (x - 1)^2 - 4$$



b. How do the functions compare to each other?

c. How do *g(x)* and *h(x)* compare to *f(x)*?

d. Look back at the functions above and identify the x-intercepts of *g(x)*. What are they?

e. What are the coordinates of the points corresponding to the x-intercepts in *g(x)* in each of the other functions? How do these coordinates compare to one another?

9. How can the transformations that occur to the function $f(x) = x^2$ be used to determine where the x-intercepts of the function's image will be?

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GO

Topic: Function Notation and Evaluating Functions

Use the given functions to find the missing values. (Check your work using a graph.)

10. $f(x) = x^2 + 4x - 12$

a. $f(0) =$ ____

b. $f(2) =$ ____ $(2)^2+4(2)-12$

c. $f(x)=0$, $x = 2, -6$

$x^2+4x-12=0$

$(x-2)(x+6)$

$2 \quad -6$

$-12: \ -2, 6$

d. $f(x) = 20$, $x = 4, -8$

$x^2+4x-12=20$

$-20 \quad -20$

$x^2+4x-32=0$

$(x-4)(x+8)$ $\quad -32: \ -4, 8$

$x = 4, -8$

11. $g(x) = (x-5)^2 + 2$

a. $g(0) =$ ____

b. $g(5) =$ ____

c. $g(x) = 0$, $x =$ ____

d. $g(x) = 16$, $x =$ ____

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12. $f(x) = x^2 - 6x + 9$

a. $f(0) =$ ____

b. $f(-3) =$ ____

c. $f(x) = 0$, $x =$ ____

d. $f(x) = 16$, $x =$ ____

13. $g(x) = (x-2)^2 - 3$

a. $g(0) =$ ____

b. $g(5) =$ ____

c. $g(x) = 0$, $x =$ ____

d. $g(x) = -3$, $x =$ ____

14. $f(x) = (x+5)^2$

a. $f(0) =$ ____

b. $f(-2) =$ ____

c. $f(x) = 0$, $x =$ ____

d. $f(x) = 9$, $x = -2, -8$

$\sqrt{(x+5)^2} = \sqrt{9}$

$x+5 = \pm 3$

$-5 \quad -5$

$x = -5 \pm 3$

$x = -2, -8$

15. $g(x) = -(x+1)^2 + 8$

a. $g(0) =$ ____

b. $g(2) =$ ____

c. $g(x) = 0$, $x =$ ____

d. $g(x) = 4$, $x =$ ____

$-(x+1)^2 + \cancel{8} = 0$

$-8 \quad -8$

$\frac{-(x+1)^2}{-1} = \frac{-8}{-1}$

$\sqrt{(x+1)^2} = \sqrt{8}$ $\quad 8: \ 4, 2$

$x + \cancel{1} = \pm 2\sqrt{2}$

$-1 \quad -1$

$x = -1 \pm 2\sqrt{2}$

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