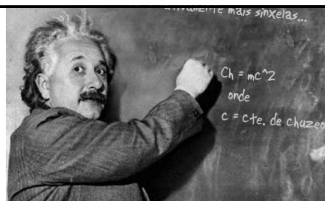
SOLVING QUADRATIC & OTHER EQUATIONS – 3.4

3.4 Radical Ideas

A Practice Understanding Task



Now that Tia and Tehani know that $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ they are wondering which form, radical form or exponential form, is best to use when working with numerical and algebraic expressions.

Tia says she prefers radicals since she understands the following properties for radicals (and there are not too many properties to remember):

If n is a positive integer greater than 1 and both a and b are positive real numbers then,

1. $\sqrt[n]{a^n} = a$ $\qquad \sqrt{x^2} = x$

2. $\sqrt[n]{ab} = \sqrt[n]{a} \cdot \sqrt[n]{b}$

3. $\sqrt[n]{\dfrac{a}{b}} = \dfrac{\sqrt[n]{a}}{\sqrt[n]{b}}$

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Tehania says she prefers exponents since she understands the following properties for exponents (and there are more properties to work with):

1. $a^m \cdot a^n = a^{m+n}$
2. $\left(a^m\right)^n = a^{mn}$
3. $(ab)^n = a^n \cdot b^n$
4. $\left(\dfrac{a}{b}\right)^n = \dfrac{a^n}{b^n}$
5. $\dfrac{a^m}{a^n} = a^{m-n}, \quad a \neq 0$
6. $a^{-n} = \dfrac{1}{a^n}$

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DO THIS: Illustrate with examples and explain, using the properties of radicals and exponents, why $a^{1/n} = \sqrt[n]{a}$ and $a^{m/n} = \left(\sqrt[n]{a}\right)^m$ are true identities.

$(a^{\frac{1}{n}})^n = (\sqrt[n]{a})^n$

$a = a$

$(2)^3 = (\sqrt[3]{2^3})^3$

$2^3 = 2^3$

$8 = 8$ ✓

Using their preferred notation, Tia might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = \sqrt[3]{x^3 \cdot x^3 \cdot x^2} = \sqrt[3]{x^3} \cdot \sqrt[3]{x^3} \cdot \sqrt[3]{x^2} = x \cdot x \cdot \sqrt[3]{x^2} = x^2 \cdot \sqrt[3]{x^2}$$

(Tehani points out that Tia also used some exponent rules in her work.)

On the other hand, Tehani might simplify $\sqrt[3]{x^8}$ as follows:

$$\sqrt[3]{x^8} = x^{8/3} = x^{2+2/3} = x^2 \cdot x^{2/3} \text{ or } x^2 \cdot \sqrt[3]{x^2}$$

For each of the following problems, simplify the expression in the ways you think Tia and Tehani might do it.

Original expression	What Tia and Tehani might do to simplify the expression:
$\sqrt{27}$ $3\sqrt{3}$ 27 = 3 · 9 = 3 · 3 · 3	Tia's method $\sqrt{27} = \sqrt{3^3} = \sqrt{3^2 \cdot 3} = \sqrt{3^2}\sqrt{3} = \boxed{3\sqrt{3}}$ ✓ Tehani's method $\sqrt{27} = (27)^{\frac{1}{2}} = (3^2 \cdot 3)^{\frac{1}{2}} = 3^{1} \cdot 3^{\frac{1}{2}} = \boxed{3\sqrt{3}}$ ✓
$\sqrt[3]{32}$ $2\sqrt[3]{4}$ 32 = 2 · 16 = 2 · 2 · 4 = 2 · 2 · 2 · 2 · 2	Tia's method $\sqrt[3]{32} = \sqrt[3]{2^3 \cdot 2^2} = \sqrt[3]{2^3} \cdot \sqrt[3]{2^2} = \boxed{2\sqrt[3]{4}}$ Tehani's method $\sqrt[3]{32} = 32^{\frac{1}{3}} = (2^3 \cdot 2^2)^{\frac{1}{3}} = 2^{1} \cdot 2^{\frac{2}{3}} = 2\sqrt[3]{2^2}$ $\boxed{2\sqrt[3]{4}}$

$\sqrt{20x^7}$	Tia's method
	Tehani's method
$\sqrt[3]{\frac{2^3 \cdot 2\, y^3}{x^6}}$ simplify $\sqrt[3]{\frac{16xy^5}{x^7y^2}}$ $16 \to 4, 4 \to 2, 2, 2, 2$	Tia's method $\frac{\cancel{\sqrt[3]{2^3}}\,\sqrt[3]{2}\,\cancel{\sqrt[3]{y^3}}}{\sqrt[3]{x^{6}}}$ $x^2 = x \cdot x$ $\boxed{\frac{2\sqrt[3]{2}\,y}{x^2}}$
	Tehani's method $\left(\frac{2^3 \cdot 2\, y^3}{x^6}\right)^{\frac{1}{3}}$ $\frac{2^{\frac{3}{3}} \cdot 2^{\frac{1}{3}}\, y^{\frac{3}{3}}}{x^{\frac{6}{3}}} = \boxed{\frac{2\sqrt[3]{2}\,y}{x^2}}$

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Tia and Tehani continue to use their preferred notation when solving equations.

For example, Tia might solve the equation $(x+4)^3 = 27$ as follows:

$$\sqrt[3]{(x+4)^3} = \sqrt[3]{27}$$
$$\sqrt[3]{(x+4)^3} = \sqrt[3]{27} = \sqrt[3]{3^3}$$
$$x + 4 = 3$$
$$x = -1$$

$\sqrt[3]{27}$ $27 \to 3, 9 \to 3, 3$ $\boxed{3}$

Tehani might solve the same equation as follows:

$$(x+4)^3 = 27$$
$$\left[(x+4)^3\right]^{\frac{1}{3}} = 27^{\frac{1}{3}} = (3^3)^{\frac{1}{3}}$$
$$x+4=3$$
$$x=-1$$

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$\pm\sqrt{9} = \pm 3$

$(x-2)^2 = 50$	Tia's method	$\sqrt{(x-2)^2} = \pm\sqrt{50}$ $x-2 = \pm\sqrt{50}$ $x-2 = \pm 5\sqrt{2}$ $x = 2 \pm 5\sqrt{2}$ $\boxed{2+5\sqrt{2}}$, $\boxed{2-5\sqrt{2}}$
	Tehani's method	$x = 2 + 5 \cdot 2^{\frac{1}{2}}, \; 2 - 5 \cdot 2^{\frac{1}{2}}$
$9(x-3)^2 = 4$	Tia's method	$\sqrt{(x-3)^2} = \pm\sqrt{\frac{4}{9}}$ $x - 3 = \frac{\sqrt{4}}{\sqrt{9}} = \pm\frac{2}{3}$ $x = 3 \pm \frac{2}{3}$ $x = 3\frac{2}{3}, \; 2\frac{1}{3}$
	Tehani's method	

SECONDARY MATH II // MODULE 3

SOLVING QUADRATICS & OTHER EQUATIONS – 3.4

3.4

READY, SET, GO! Name Period Date

Start 3.4 HW

READY

Topic: Standard form or Quadratic form

In each of the quadratic equations, $ax^2 + bx + c = 0$ **identify the values of a, b and c .**

1. $1x^2 + 3x + 2 = 0$

   a = 1
   b = 3
   c = 2

2. $2x^2 + 3x + 1 = 0$

   a =
   b =
   c =

3. $x^2 - 4x - 12 = 0$

   a =
   b =
   c =

Write each of the quadratic expressions in factored form.

4. $x^2 + 3x + 2$

$(x+1)(x+2)$

5. $2x^2 + 3x + 1$

$(2x+1)(x+1)$

	$2x$	$+1$
x	$2x^2$	$1x$
$+1$	$2x$	1

6. $x^2 - 4x - 12$

7. $x^2 - 3x + 2$

8. $x^2 - 5x - 6$

9. $x^2 - 4x + 4$

10. $x^2 + 8x - 20$

11. $x^2 + x - 12$

12. $x^2 - 7x + 12$

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	Radical Form	Exponential Form
13.	$\sqrt[3]{5^2}$	
14.		$16^{\frac{3}{4}}$
15.	$\sqrt[3]{5^7 \cdot 3^5}$	
16.	$\sqrt[3]{9^2} \cdot \sqrt[3]{9^4}$	$9^{\frac{2}{3}} \cdot 9^{\frac{4}{3}}$
17.	$\sqrt[5]{x^{13}y^{21}}$	
18.	$\sqrt[3]{27a^5b^2}$	
19.	$\sqrt[5]{\dfrac{32x^{13}}{243y^{15}}}$	
20.		$9^{\frac{3}{2}}s^{\frac{6}{3}}t^{\frac{1}{2}}$

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R √ = ±

Solve the equations below, use radicals or rational exponents as needed.

21. $\sqrt[4]{(x+5)^4} = \sqrt[4]{81}$ $81 < 9, 9$; $9 < 3, 3$; $9 < 3, 3$

$x + 5 = \sqrt[4]{3^4}$

$x + 5 = \pm 3$

$\quad -5 \quad -5$

$x = -5 + 3, \ -5 - 3$

$\boxed{x = -2, -8}$

22. $2(x-7)^5 + 3 = 67$

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GO

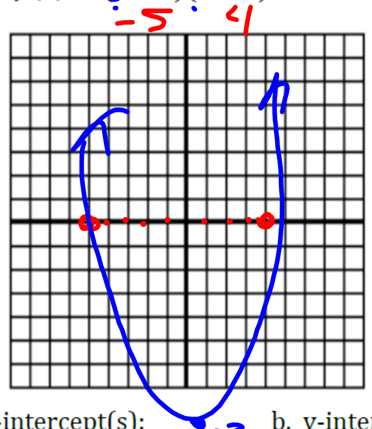
Topic: x-intercepts and y-intercepts for linear, exponential and quadratic functions

Given the function, find the x-intercept (s) and y-intercept if they exist and then use them to graph a sketch of the function.

$\sim x^2$

23. $f(x) = (x+5)(x-4)$

$-5 \quad 4$



a. x-intercept(s): $-5, 4$

b. y-intercept: -20

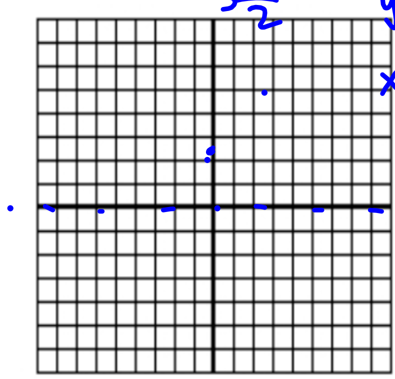
$(5)(-4)$
$= -20$

24. $g(x) = 5(2^{x+1})$ expo

$\frac{5 \cdot 2^x}{2}$

yint $\rightarrow x = 0$

xint $\rightarrow y = 0$



a. x-intercept(s):

b. y-intercept:

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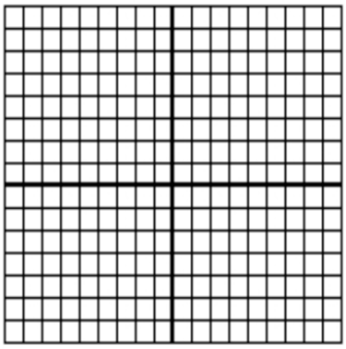
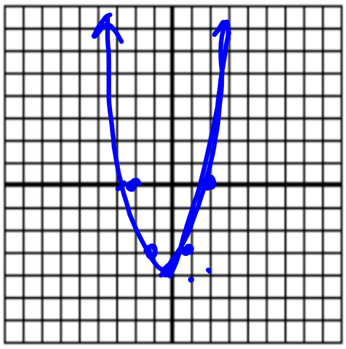
25. $h(x) = -2(x + 3)$ $-x^1$



a. x-intercept(s): b. y-intercept:

26. $k(x) = x^2 - 4$ $-x^2$



a. x-intercept(s): b. y-intercept: