

3.3 More Interesting



A Solidify Understanding Task

Carlos now knows he can calculate the amount of interest earned on an account in smaller increments than one full year. He would like to determine how much money is in an account each month that earns 5% annually with an initial deposit of \$300.

He starts by considering the amount in the account each month during the first year. He knows that by the end of the year the account balance should be \$315, since it increases 5% during the year.

- Complete the table showing what amount is in the account each month during the first twelve months.

$5\% \text{ of } 300 = 15$

Time	0	1/12	2/12	3/12	4/12	5/12	6/12	7/12	8/12	9/12	10/12	11/12	12/12
Account balance	\$300	301.22	302.47	303.68	304.86	306.10	307.41	308.68	309.92	311.18	312.45	313.70	\$315

$300 \cdot 1.0007$
 $300 \cdot 1.05$
 $300 \cdot 1.05^{1/12}$

$100\% + 5\% = 105\% \rightarrow 1.05 = 1.0007$

- What number did you multiply the account by each month to get the next month's balance?

Carlos knows the exponential equation that gives the account balance for this account on an annual basis is $A = 300(1.05)^t$. Based on his work finding the account balance each month, Carlos writes the following equation for the same account: $A = 300(1.05^{1/12})^{12t}$.

- Verify that both equations give the same results. Using the properties of exponents, explain why these two equations are equivalent.

$$\checkmark 300(1.05)^t = 300(1.05^{1/12})^{12t} \quad \frac{12t}{12} = t$$

- What is the meaning of the $12t$ in this equation?

12 months.

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Carlos shows his equation to Clarita. She suggests his equation could also be approximated by $A = 300(1.004)^{12t}$, since $(1.05)^{1/12} \approx 1.004$. Carlos replies, "I know the 1.05 in the equation $A = 300(1.05)^t$ means I am earning 5% interest annually, but what does the 1.004 mean in your equation?"

- Answer Carlos' question. What does the 1.004 mean in $A = 300(1.004)^{12t}$?

rate per month.

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The properties of exponents can be used to explain why $[(1.05)^{\frac{1}{12}}]^{12t} = 1.05^t$. Here are some more examples of using the properties of exponents with rational exponents. For each of the following, simplify the expression using the properties of exponents, and explain what the expression means in terms of the context.

6. $(1.05)^{\frac{1}{12}} \cdot (1.05)^{\frac{1}{12}} \cdot (1.05)^{\frac{1}{12}} = 1.05^{\frac{1}{12} + \frac{1}{12} + \frac{1}{12}} = 1.05^{\frac{3}{12}} = 1.05^{\frac{1}{4}}$
 3 months interest rate, quarter of a year
7. $[(1.05)^{\frac{1}{12}}]^6 = 1.05^{\frac{6}{12}} = 1.05^{\frac{1}{2}}$
 6 months, 1/2 of a year
8. $(1.05)^{-\frac{1}{12}} \Rightarrow 1.05^{\frac{1}{12}}$
 back one month
9. $(1.05)^2 \cdot (1.05)^{\frac{1}{4}} = 1.05^{2 + \frac{1}{4}} = 1.05^{\frac{8}{4} + \frac{1}{4}} = 1.05^{\frac{9}{4}}$
 2 1/4 years of interest
10. $\frac{(1.05)^2}{(1.05)^{\frac{1}{2}}} = 1.05^{2 - \frac{1}{2}} = 1.05^{\frac{4}{2} - \frac{1}{2}} = 1.05^{\frac{3}{2}}$
 1.5 years interest.
11. Use $[(1.05)^{\frac{1}{12}}]^{12} = 1.05$ to explain why $(1.05)^{\frac{1}{12}} = \sqrt[12]{1.05}$
 $(1.05)^{\frac{1}{12}} = (\sqrt[12]{1.05})^{12} = 1.05 = 1.05 \checkmark$

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READY, SET, GO!

Name

Period

Date

READY

Starter: 'ready'

Topic: Meaning of Exponents

In the table below there is a column for the exponential form, the meaning of that form, which is a list of factors and the standard form of the number. Fill in the form that is missing.

Exponential form	List of factors	Standard Form
5^3	$5 \cdot 5 \cdot 5$	125
1a.	$7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7 \cdot 7$	b.
2.	2^{10}	a.
3a.	b.	81
4.	11^5	a.
5a.	$3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3 \cdot 3$	b.
6a.	b.	625

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Provide at least three other equivalent forms of the exponential expression. Use rules of exponents such as $3^5 \cdot 3^6 = 3^{11}$ and $(5^2)^3 = 5^6$ as well as division properties and others.

	1 st Equivalent Form	Equivalent Form	Equivalent Form
7. $2^{10} =$	$x \cdot x^b$ $2^6 \cdot 2^4 = 10 \checkmark$	$(x^a)^b$ $(2^2)^5 = 10 \checkmark$	x^a / x^b $2^{12} / 2^2 = 10 \checkmark$
8. $3^7 =$			
9. $13^{-8} =$			
10. $7^{\frac{1}{3}} =$	$7^{\frac{1}{6}} \cdot 7^{\frac{1}{6}} = \frac{2}{6} = \frac{1}{3}$	$(7^{\frac{1}{2}})^2 = \frac{2}{6} = \frac{1}{3}$	$7^{\frac{2}{3}} / 7^{\frac{1}{3}} = \frac{1}{3}$
11. $5^1 =$			

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SET

Topic: Finding equivalent expressions and functions

Determine whether all three expressions in each problem below are equivalent. Justify why or why they are not equivalent.

12. $5(3^{x-1})$ $15(3^{x-2})$ $5(3^x)^{\frac{5}{3}(3^x)}$ *equivalent.*

13. $64(2^{-x})$ $\frac{64}{2^x}$ $64(\frac{1}{2})^x$

14. $3(x-1)+4$ $3x-1$ $3(x-2)+7$

15. $50(2^{2x+2})$ $25(2^{2x+1})$ $50(4^x)$

16. $30(1.05^x)$ $30(1.05^{\frac{1}{7}})^{7x}$ $30(1.05^{\frac{x}{2}})^2$ *equivalent.*

17. $20(1.1^x)$ $20(1.1^{-1})^{-1x}$ $20(1.1^{\frac{1}{5}})^{5x}$

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GO

Topic: Using rules of exponents

Simplify each expression. Your answer should still be in exponential form.

18. $7^3 \cdot 7^5 \cdot 7^2$

19. $(3^4)^5$

20. $(5^3)^4 \cdot 5^7$

21. $x^3 \cdot x^5$

22. x^{-b}

23. $x^a \cdot x^b$

24. $(x^a)^b$

25. $\frac{y^a}{y^b}$ y^{a-b}

26. $\frac{(y^a)^c}{y^b} = y^{ac-b}$

27. $\frac{(3^4)^6}{3^7}$

28. $\frac{r^5 s^3}{r s^2}$

$\frac{r \cdot r \cdot r \cdot r \cdot r \cdot \cancel{r} \cdot \cancel{s} \cdot \cancel{s} \cdot \cancel{s}}{\cancel{r} \cdot \cancel{s} \cdot \cancel{s}}$

$r^4 s$

29. $\frac{5 \cdot 12 \cdot 2^0}{8 \cdot y^9}$

$\frac{y^4}{x^3}$