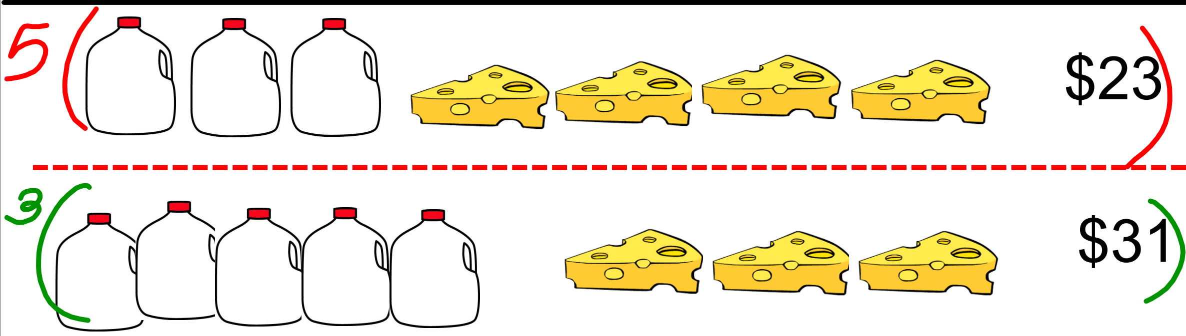
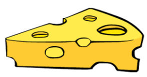
$$3x + 4y = 23$$

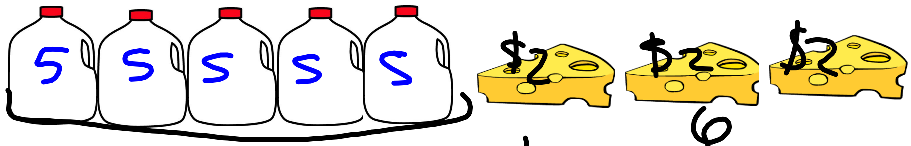
$$5x + 3y = 31$$



$$\begin{aligned} \cancel{15x} + 20y &= 115 \\ -\cancel{15x} - 9y &= -93 \\ \hline 11y &= 22 \end{aligned}$$

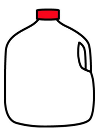

= $2

Now we know cheese is $2, find the cost of milk.



25 + 6

$31
-6
25


= 5

3.13H All Systems Go!

A Solidify Understanding Task

Carlos likes to buy supplies for *Curbside Rivalry* at the *All a Dollar Paint Store* where the price of every item is a multiple of $1. This makes it easy to keep track of the total cost of his purchases. Clarita is worried that items at *All a Dollar Paint Store* might cost more, so she is going over the records to see how much Carlos is paying for different supplies. Unfortunately, Carlos has once again forgotten to write down the cost of each item he purchased. Instead, he has only recorded what he purchased and the total cost of all of the items.

Carlos and Clarita are trying to figure out the cost of a gallon of paint, the cost of a paintbrush, and the cost of a roll of masking tape based on the following purchases:

$2g + 1t = 30$

Week 1: Carlos bought 2 gallons of paint and 1 roll of masking tape for $30.

$1g + \cancel{4b} = 20 - 4b$

Week 2: Carlos bought 1 gallon of paint and 4 brushes for $20.

$\cancel{2b} + 1t = 10 - 2b$

Week 3: Carlos bought 2 brushes and 1 roll of masking tape for $10.

1. Determine the cost of each item using whatever strategy you want. Show the details of your work so that someone else can follow your strategy.

Substitution

$t = 10 - 2b$

$g = 20 - 4b$

g b t

$2g + t = 30$

$2(20-4b) + (10-2b) = 30$

$40 - 8b + 10 - 2b$

$\cancel{50} - 10b = 30$

$\cancel{-50} \quad\quad -50$

$\frac{\cancel{-10}b}{\cancel{-10}} = \frac{-20}{-10}$

$\boxed{b = 2}$

$t = 10 - 2(2)$

$\boxed{t = 6}$

$g = 20 - 4(2)$

$\boxed{g = 12}$

In the MVP Secondary Math I tasks *To Market with Matrices* and *Solving Systems with Matrices* you learned how to solve systems of equations involving two equations and two unknown quantities using row reduction of matrices. You may want to review those two tasks before continuing.

3. Modify the "row reduction of matrices" strategy so you can use it to solve Carlos and Clarita's system of equations using row reduction. What modifications did you have to make, and why?

$$\begin{array}{cccc} g & b & t & \$ \end{array}$$

$$\begin{bmatrix} 2 & 0 & 1 & 30 \\ 1 & 4 & 0 & 20 \\ 0 & 2 & 1 & 10 \end{bmatrix}$$

$R_1 - R_3 \rightarrow R_1$

$$\begin{bmatrix} 2 & -2 & 0 & 20 \\ 1 & 4 & 0 & 20 \\ 0 & 2 & 1 & 10 \end{bmatrix}$$

$2R_1 + R_2 \rightarrow R_1$

$$\begin{bmatrix} 5 & 0 & 0 & 60 \\ 1 & 4 & 0 & 20 \\ 0 & 2 & 1 & 10 \end{bmatrix}$$

$\frac{1}{5}R_1$

$$\begin{bmatrix} 1 & 0 & 0 & 12 \\ 1 & 4 & 0 & 20 \\ 0 & 2 & 1 & 10 \end{bmatrix}$$

$R_1 - R_2 \rightarrow R_2$

$$\begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & -4 & 0 & -8 \\ 0 & 2 & 1 & 10 \end{bmatrix} \div 4$$

$R_2 \div -4$

$$\begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 2 \\ 0 & 2 & 1 & 10 \end{bmatrix}$$

$R_3 - 2R_2 \rightarrow R_3$

$$\begin{bmatrix} 1 & 0 & 0 & 12 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 6 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \text{ goal}$$

$$\boxed{\begin{aligned} p &= 12 \\ b &= 2 \\ t &= 6 \end{aligned}}$$

Multiply the follow pairs of matrices:

a.
$$\begin{array}{c} R_1 \\ R_2 \\ R_3 \end{array}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{array}{c} \begin{array}{ccc} C_1 & C_2 & C_3 \end{array} \\ \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} (1\cdot2)+(0\cdot1)+(0\cdot0) & (1\cdot0)+(0\cdot4)+(0\cdot2) & (1\cdot1)+(0\cdot0)+(0\cdot1) \\ 2 & 0 & 1 \\ 0+1+0 & 0+4+0 & 0+0+0 \\ 1 & 4 & 0 \\ 0+0+0 & 0+0+2 & 0+0+1 \\ 0 & 2 & 1 \end{bmatrix}$$

$$\boxed{\text{example } 1 \cdot 5 = 5}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

The matrix was multiplied by the Identity matrix, so you just get the same matrix as the result.

b.
$$\begin{bmatrix} 0.4 & 0.2 & -0.4 \\ -0.1 & 0.2 & 0.1 \\ 0.2 & -0.4 & 0.8 \end{bmatrix} \cdot \begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix} = \begin{bmatrix} .8+.2+0 & 0+.8-.8 & .4+0-.4 \\ 1 & 0 & 0 \\ -.2+.2+0 & 0+.8+.2 & 0 \\ 0 & 1 & \\ 0 & 0 & 1 \end{bmatrix}$$

$$5 \cdot \frac{1}{5} = 1$$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The results was the identity matrix, so our matrices were $\boxed{\text{inverses}}$

5. What property is illustrated by the multiplication in question 4a?

6. What property is illustrated by the multiplication in question 4b?

7. Rewrite the following system of equations, which represents Carlos and Clarita's problem, as a matrix equation in the form **AX = B** where **A**, **X** and **B** are all matrices.

$$\begin{array}{c} g \quad b \quad t \\ 2g + 0b + 1t = 30 \\ 1g + 4b + 0t = 20 \\ 0g + 2b + 1t = 10 \end{array}$$

$$\begin{bmatrix} 2 & 0 & 1 \\ 1 & 4 & 0 \\ 0 & 2 & 1 \end{bmatrix} \begin{bmatrix} g \\ b \\ t \end{bmatrix} = \begin{bmatrix} 30 \\ 20 \\ 10 \end{bmatrix}$$

8. Solve your matrix equation by using multiplication of matrices. Show the details of your work so that someone else can follow it.

You were able to solve this equation using matrix multiplication because you were given the inverse of matrix **A.** Unlike 2×2 matrices, where the inverse matrix can be easily found by hand using the methods described in *More Arithmetic of Matrices*, the inverses of $n \times n$ in general can be difficult to find by hand. In such cases, we will use technology to find the inverse matrix so that this method can be applied to all linear systems involving n equations and n unknown quantities.

SOLVING QUADRATICS & OTHER EQUATIONS – 3.13H **3.13H**

READY, SET, GO! Name Period Date

READY

Topic: Rational Exponents Review and methods for solving quadratics

Write each exponential expression in radical form.

1. $10^{\frac{3}{2}}$ $\sqrt[2]{10^3}$ or $(\sqrt[2]{10})^3$

2. $x^{\frac{1}{5}}$

3. $3n^{\frac{1}{3}}$

4. $6^{\frac{2}{7}}$

5. $7^{\frac{5}{3}}$

6. $t^{\frac{4}{5}}$

Write each radical expression in exponential form.

7. $(\sqrt[5]{3})$ $3^{\frac{1}{5}}$

8. $(\sqrt[6]{7a})^5$

9. $\sqrt{x^3}$

10. $\sqrt[3]{n^5}$

11. $(\sqrt[y]{n})^x$

12. $\sqrt[p]{n^q}$

Explain each strategy for solving quadratic equations and explain the circumstances in which the strategy is most efficient.

13.
Graphing

14.
Factoring

15.
Completing the square

16. What other strategies do you know for solving quadratic equations? When would you use them?

A7 notes

SET

Topic: Solving systems with three unknowns.

Solve the system of equations using matrices. Create a matrix equation for the system of equations that can be used to find the solution. Then find the inverse matrix and use it to solve the system.

on seperate sheet of paper! (Long) →

17. $\begin{cases} 2x - 4y + z = 0 \\ 5x - 4y - 5z = 12 \\ 4x + 4y + z = 24 \end{cases}$

18. $\begin{cases} x + 2y + 5z = -15 \\ x + y - 4z = 12 \\ x - 6y + 4z = -12 \end{cases}$

19. $\begin{cases} 4p + q - 2r = 5 \\ -3p - 3q - 4r = -16 \\ 4p - 4q + 4r = -4 \end{cases}$

20. $\begin{cases} -6x - 4y + z = -20 \\ -3x - y - 3z = -8 \\ -5x + 3y + 6z = -4 \end{cases}$

17) $\begin{bmatrix} 2 & -4 & 1 & 0 \\ 5 & -4 & -5 & 12 \\ 4 & 4 & 1 & 24 \end{bmatrix}$

goal $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

gaussian elimination.

$R_1 + R_3 \rightarrow R_1$

$\begin{bmatrix} 6 & 0 & 2 & 24 \\ 5 & -4 & -5 & 12 \\ 4 & 4 & 1 & 24 \end{bmatrix}$

$R_2 + R_3 \rightarrow R_3$

$2(6 \; 0 \; 2 \; 24)$

$\begin{bmatrix} 6 & 0 & 2 & 24 \\ 5 & -4 & -5 & 12 \\ 9 & 0 & -4 & 36 \end{bmatrix}$

$2R_1 + R_3 \rightarrow R_1$

$(21 \; 0 \; 0 \; 84) \div 21$

$(1 \; 0 \; 0 \; 4) \cdot -9$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 5 & -4 & -5 & 12 \\ 9 & 0 & -4 & 36 \end{bmatrix} +$

$-9R_1 + R_3 \rightarrow R_3$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 5 & -4 & -5 & 12 \\ 0 & 0 & -4 & 0 \end{bmatrix} \div 4$

$-5(1 \; 0 \; 0 \; 4)$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 5 & -4 & -5 & 12 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

$-5R_1 + R_2 \rightarrow R_2$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & -4 & -5 & -8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ + $(0 \; 0 \; 1 \; 0) \cdot 5$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & -4 & 0 & -8 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $(0 \; -4 \; 0 \; -8) \div -4$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix} \rightarrow$ $x = 4$, $y = 2$, $z = 0$

SET

A5 notes

Topic: Solving systems with three unknowns.

Solve the system of equations using matrices. Create a matrix equation for the system of equations that can be used to find the solution. Then find the inverse matrix and use it to solve the system.

$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ goal

17. $\begin{cases} 2x - 4y + z = 0 \\ 5x - 4y - 5z = 12 \\ 4x + 4y + z = 24 \end{cases}$

$\begin{matrix} 2 & -4 & 1 & 0 \\ 5 & -4 & -5 & 12 \\ 4 & 4 & 1 & 24 \end{matrix}$

$r_1 + r_3 \rightarrow r_3$

$\begin{matrix} 2 & -4 & 1 & 0 \\ 5 & -4 & -5 & 12 \\ 9 & 0 & -4 & 36 \end{matrix}$

$\begin{bmatrix} -3 & 0 & 6 & -12 \\ 5 & -4 & -5 & 12 \\ 9 & 0 & -4 & 36 \end{bmatrix}$

$3R_1 + r_3 \rightarrow r_3$

$\begin{bmatrix} -3 & 0 & 6 & -12 \\ 5 & -4 & -5 & 12 \\ 0 & 0 & 14 & 0 \end{bmatrix}$

$\begin{bmatrix} -3 & 0 & 6 & -12 \\ 5 & -4 & -5 & 12 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\times 6$

$-6R_3 + R_1 \rightarrow R_1$

$\begin{bmatrix} -3 & 0 & 0 & -12 \\ 5 & -4 & -5 & 12 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\div -3$

$\begin{matrix} 1 & 0 & 0 & 4 \\ 5 & -4 & -5 & 12 \\ 0 & 0 & 1 & 0 \end{matrix}$ $5+$

$R_2 + 5R_3 \rightarrow R_2$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 5 & -4 & 0 & 12 \\ 0 & 0 & 1 & 0 \end{bmatrix}$ $\cdot 5$

$-5R_1 + R_2 \rightarrow R_2$

$\begin{matrix} 1 & 0 & 0 & 4 \\ 0 & -4 & 0 & -8 \\ 0 & 0 & 1 & 0 \end{matrix}$ $\div -4$

$\begin{bmatrix} 1 & 0 & 0 & 4 \\ 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

18. $\begin{cases} x + 2y + 5z = -15 \\ x + y - 4z = 12 \\ x - 6y + 4z = -12 \end{cases}$

19. $\begin{cases} 4p + q - 2r = 5 \\ -3p - 3q - 4r = -16 \\ 4p - 4q + 4r = -4 \end{cases}$

20. $\begin{cases} -6x - 4y + z = -20 \\ -3x - y - 3z = -8 \\ -5x + 3y + 6z = -4 \end{cases}$

GO

Topic: Solving Quadratics

Solve each of the quadratics below using an appropriate and efficient method.

21. $x^2 - 5x = -6$

22. $3x^2 - 5 = 0$

23. $5x^2 - 10 = 0$

24. $x^2 + 1x - 30 = 0$

25. $x^2 + 2x = 48$

26. $x^2 - 3x = 0$