3.10 iNumbers

A Practice Understanding Task

In order to find solutions to all quadratic equations, we have had to extend the number system to include complex numbers.

$-3\frac{7}{3}$ -2 -1 0 1 2 2.5 3 4 $\pm i$

$\sqrt{2}$ e π

Real Numbers

Natural

Whole

Integers

Rationals

Irrationals π

Complex Numbers i

Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least three examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

Conjecture #1: The sum of two integers is [always, sometimes, never] an integer.

$-2 + -4 =$ ✓

$2 + 1 =$ ✓

Conjecture #2: The sum of two rational numbers is [always, sometimes, never] a rational number.

$\frac{0}{1} = 0$ $\quad 7\frac{1}{2} + \frac{3}{2} = \frac{9}{1}$ ✓

$\frac{1}{2} + \left(-\frac{1}{2}\right) = \frac{0}{1}$ ✓

Conjecture #3: The sum of two irrational numbers is [always, sometimes, never] an irrational number.

$\pi - \pi = 0$ whole!

$\pi + \sqrt{2}$ ...

$\sqrt{2} - \sqrt{2} = 0$

$\sqrt{2} + (1 - \sqrt{2}) = 1$

1.41... ...59...

Conjecture #4: The sum of two real numbers is [always, sometimes, never] a real number.

$2 + 1 = 3$

$\sqrt{2} + \pi = \ldots$ no imaginary

$-\sqrt{2} + -\sqrt{2}$

Conjecture #5: The sum of two complex numbers is [always, sometimes, never] a complex number.

$2i + 2 = 2i + 2$

$\cancel{2i} - \cancel{2i} + 3 = 3 + 0i$

Conjecture #6: The product of two integers is [always, sometimes, never] an integer.

$-5 \cdot 5 \quad -25$ ✓

$2 \cdot 3 = 6$ ✓

Conjecture #7: The quotient of two integers is [always, sometimes, never] an integer.

$\frac{5}{10} = \frac{1}{2}$ ✗ $\quad \frac{10}{5} = 2$ ✓

Conjecture #8: The product of two rational numbers is [always, sometimes, never] a rational number.

$\frac{1}{2} \times \frac{2}{3} = \frac{2}{6} = \frac{1}{3}$ ✓

$\frac{2}{3} \times \frac{3}{2} = 1$ ✓

Conjecture #9: The quotient of two rational numbers is [always, sometimes, never] a rational number.

$\frac{\frac{1}{2}}{\frac{1}{2}}$ $2 = \frac{1}{1}$ ✓ $\frac{7}{43} = \ldots$ ✓

$\frac{7}{0}$ undefined

Conjecture #10: The product of two irrational numbers is [always, sometimes, never] an irrational number.

$\sqrt{2}\cdot\sqrt{2} = 2$ ✗

$\sqrt{6} = \sqrt{2}\sqrt{3}$

$\sqrt{14} = \sqrt{2}\cdot\sqrt{7}$

$\sqrt{2}\cdot\sqrt{7} = (2\cdot 7)^{\frac{1}{2}}$ ✓

$\sqrt{14}$ ✓

Conjecture #11: The product of two real numbers is [always, sometimes, never] a real number.

$2 \cdot 5 = 10$ ✓

$\sqrt{2}\sqrt{2} = 2$ ✓

$\frac{\pi}{3}\cdot\frac{3}{\pi} = 1$ ✓

$\pi \cdot \pi =$ ✓

Conjecture #12: The product of two complex numbers is [always, sometimes, never] a complex number.

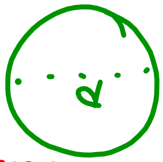
$2 \cdot 2i = 4i$ ✓

$2 \cdot \frac{1}{2} \cdot \underbrace{i \cdot i}_{-1} = 1$ ✓ ← no i's!? yes!

$\mathbb{C} \cdot \mathbb{C} = \underline{\mathbb{C}}$ always.

because everything is complex

13. The ratio of the circumference of a circle to its diameter is given by the irrational number π. Can the diameter of a circle and the circumference of the same circle both be rational numbers? Explain why or why not.



$\frac{C}{d} = \pi$

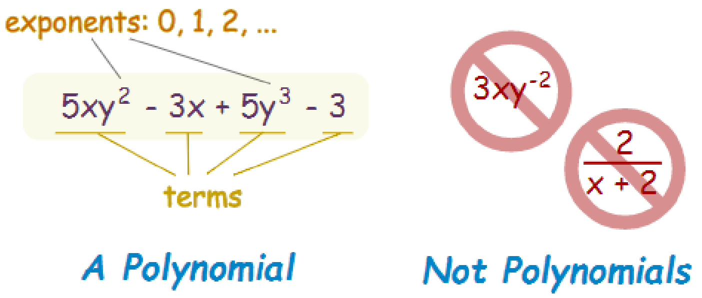
$\underline{10}$

$C = d\pi$

$\frac{10}{\pi} = d$

X circumference or diameter would have to be irrational

Polynomial or Not?



These **are** polynomials:

- $3x$
- $x - 2$
- $-6y^2 - (\frac{7}{9})x$
- $3xyz + 3xy^2z - 0.1xz - 200y + 0.5$
- $512v^5 + 99w^5$
- 5
- 0 , it's a polynomial that has 0 for all the coefficients, 0x+0

(Yes, "5" is a polynomial, **one term is allowed**, and it can even be just a constant!)

And these are **not** polynomials

- $3xy^{-2}$ is not, because the exponent is "-2" (exponents can only be 0,1,2,...)
- **2/(x+2)** is not, because dividing by a variable is not allowed
- **1/x** is not either
- **√x** is not, because the exponent is "½" (see fractional exponents)

But these **are** allowed:

- **x/2 is allowed**, because you can divide by a constant
- also **3x/8** for the same reason
- **√2** is allowed, because it is a constant (= 1.4142...etc)

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The Arithmetic of Polynomials

In the task *To Be Determined . . .* we defined polynomials to be expressions of the following form:

$$a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots a_{n-3}x^3 + a_{n-2}x^2 + a_{n-1}x + a_n$$

where all of the exponents are positive integers and all of the coefficients $a_0 \ldots a_n$ are constants.

Do the following for each of the problems below:

- Choose the best word to complete each conjecture.
- After you have made a conjecture, create at least three examples to show why your conjecture is true.
- If you find a counter-example, change your conjecture to fit your work.

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Conjecture #P1: The sum of two polynomials is [always, sometimes, never] a polynomial.

$(x^2+x+1)+(2x^2+4x+6)$ ✓

$\cancel{x^2}+\cancel{x}+\cancel{1}+(-\cancel{x^2}-\cancel{x}-\cancel{1})=0$ ✓ poly

Conjecture #P2: The difference of two polynomials is [always, sometimes, never] a polynomial.

$\cancel{x^2}+2x+8-(\cancel{x^2}-x-1) = 3x+9$ ✓

$+x+1$

$(x+1)-(x+1)=0$ ✓

Conjecture #P3: The product of two polynomials is [always, sometimes, never] a polynomial.

$(x+3)(x+2)=x^2+5x+6$ ✓

$(x^2+x+3)(x^2+2x+2)=x^4\ldots$ ✓

$-2x\cdot 2x=-4x^2$ ✓

SECONDARY MATH II // MODULE 3
SOLVING QUADRATICS & OTHER EQUATIONS – 3.10

3.10

READY, SET, GO! Name Period Date

Extra Credit #1&2 if done well

READY

Topic: Attributes of quadratics and other functions

1. Summarize what you have learned about quadratic functions to this point. In addition to your written explanation provide graphs, tables and examples to illustrate what you know.

2. In prior work you have learned a great deal about both linear and exponential functions. Compare and contrast linear and exponential functions with quadratic functions. What similarities if any are there and what differences are there between linear, exponential and quadratic functions?

SET

Topic: Operations on different types of numbers

3. The Natural numbers, ℕ, are the numbers that come naturally or the counting numbers. As any child first learns numbers, they learn 1, 2, 3, ... What operations on the Natural numbers would cause the need for other types of numbers? What operation on Natural numbers create a need for Integers or Rational numbers and so forth. (Give examples and explain.)

subtraction = $3-4=-1$ ← integers.

divisions = $\frac{1}{2}$ → rational

In each of the problems below use the given items to determine whether or not it is possible *always*, *sometimes* or *never* to create a new element* **that is in the desired set.**

4. Using the operation of addition and elements from the Integers, $\mathbb{Z}$, [always, sometimes, never] an element of the Irrational numbers, $\overline{\mathbb{Q}}$, will be created. Explain.

5. Consider the equation $a - b = c$, where $a \in \mathbb{N}$ and $b \in \mathbb{N}$, c will be an Integer, $\mathbb{Z}$ [always, sometimes, never]. Explain.

6. Consider the equation $a \div b = c$, where $a \in \mathbb{Z}$ and $b \in \mathbb{Z}$, then is $c \in \mathbb{Z}$ [sometimes, always, never]. Explain.

**The numbers in any given set of numbers may be referred to as elements of the set. For example, the Rational number set, $\mathbb{Q}$, contains elements or numbers that can be written in the form $\frac{a}{b}$, where a and b are integer values ($b \neq 0$).*

7. Using the operation of subtraction and elements from the Irrationals, $\overline{\mathbb{Q}}$, an element of the Irrational numbers, $\overline{\mathbb{Q}}$, will be created [always, sometimes, never]. Explain.

8. If two Complex numbers, $\mathbb{C}$, are subtracted the result will [always, sometimes, never] be a Complex number, $\mathbb{C}$. Explain.

GO

Topic: Solving all types of Quadratic Equations, Simplifying Radicals

Make a prediction as to the nature **of the solutions for each quadratic (Real, Complex, Integer, etc.) then solve each of the quadratic equations below using an appropriate and efficient method. Give the solutions and compare to your prediction.**

category

9. $-5x^2 + 3x + 2 = 0$

Prediction: $\mathbb{C}$

discriminant

$\sqrt{b^2-4ac} \rightarrow \sqrt{+}\ \mathbb{R}$

$\sqrt{-}\ \mathbb{C}$

Solutions:

10. $x^2 + 3x + 2 = 0$

Prediction: $\mathbb{C}$

Solutions:

11. $x^2 + 3x - 12 = 0$

Prediction: $\mathbb{C}$

Solutions:

12. $\overset{a}{4x^2} - \overset{b}{19x} - \overset{c}{5} = 0$

-20, -19; $\frac{-5}{1}$, $\frac{1}{4}$

Prediction: $\mathbb{C}$

$(x-5)(4x+1)$

$5, -\frac{1}{4}$

Solutions: $x = \frac{19 \pm \sqrt{(19)^2 - 4(4)(-5)}}{2(4)}$

$\frac{19 \pm \sqrt{361 + 80}}{8}$

$\frac{19 \pm \sqrt{441}}{8}$ $\sqrt{21 \cdot 21}$

$\frac{19 \pm 21}{8}$ $\boxed{x = 5, -\frac{1}{4}}$

Start 3.10 Homework

Simplify each of the radical expressions. Use rational exponents if desired.

13. $\sqrt[4]{81x^8y^{12}}$ $\boxed{=3\cdot x^2y^3}$

$\sqrt[4]{81}\cdot\sqrt[4]{x^8}\;\sqrt[4]{y^{12}}$

9 9

3 3 3 3 $x^{\frac{8}{4}}$ $y^{\frac{12}{4}}$

14. $\sqrt{\frac{a^7b^{10}}{a^3}}$

15. $\sqrt[5]{625x^{12}}$

16. $(\sqrt{n})^5$

17. $\sqrt[3]{-27}$ odd

$\sqrt[3]{27}$ i $(-3)(-3)(3)$

-27

-3 9

3 3

$=-3$

18. $(\sqrt{8})(\sqrt{3})(2)$

2 4

2 2

$2\sqrt{2}\cdot 3\cdot 2$

$\boxed{12\sqrt{2}}$

#17, not imaginary!!!!

the 3rd root of -27 can be -3, not problem!

Fill in the table so each expression is written in radical form and with rational exponents.

	Radical Form	Exponential Form
19.	$\sqrt[4]{8^3}$	
20.		$256^{\frac{3}{4}}$
21.	$\sqrt[4]{2^7 \bullet 4^5}$	
22.		$16^{\frac{3}{2}} \bullet 4^{\frac{1}{2}}$
23.	$\sqrt[10]{x^{23}y^{31}}$	
24.	$64^{\frac{1}{5}}$ $\sqrt[5]{64a^9b^{18}}$	$2^{3/2}\ a^{\frac{9}{5}}\ b^{\frac{18}{5}}$

16 4

4 4 2

2 2 2 2 2 2

$2\sqrt{2}$

$2^1 \cdot 2^{1/2}$

$2^{(1+\frac{1}{2})}$

$2^{\frac{3}{2}}$