2.3 Building the Perfect Square

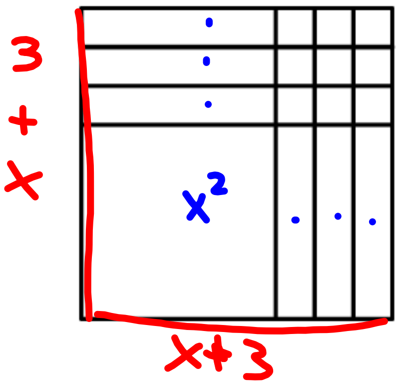
A Develop Understanding Task

Quadratic Quilts

Optima has a quilt shop where she sells many colorful quilt blocks for people who want to make their own quilts. She has quilt designs that are made so that they can be sized to fit any bed. She bases her designs on quilt squares that can vary in size, so she calls the length of the side for the basic square x, and the area of the basic square is the function $A(x) = x^2$. In this way, she can customize the designs by making bigger squares or smaller squares.

1. If Optima adds 3 inches to the side of the square, what is the area of the square?

When Optima draws a pattern for the square in problem #1, it looks like this:



$x^2 + 6x + 9$

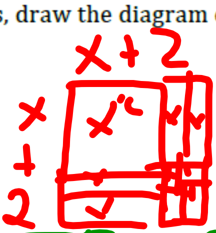
$(x+3)^2$

2. Use both the diagram and the equation, $A(x) = (x+3)^2$ to explain why the area of the quilt block square, $A(x)$, is also equal to $x^2 + 6x + 9$.

The customer service representatives at Optima's shop work with customer orders and write up the orders based on the area of the fabric needed for the order. As you can see from problem #2 there are two ways that customers can call in and describe the area of the quilt block. One way describes the length of the sides of the block and the other way describes the areas of each of the four sections of the block.

For each of the following quilt blocks, draw the diagram of the block and write two equivalent equations for the area of the block.

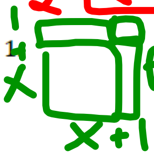
3. Block with side length: $x + 2$.



$A = x^2 + 4x + 4$

$(x+2)^2$

4. Block with side length: $x + 1$



$A = x^2 + 2x + 1$

$(x+1)^2$

5. What patterns do you notice when you relate the diagrams to the two expressions for the area?

$\ell \cdot w = A$

6. Optima likes to have her little dog, Clementine, around the shop. One day the dog got a little hungry and started to chew up the orders. When Optima found the orders, one of them was so chewed up that there were only partial expressions for the area remaining. Help Optima by completing each of the following expressions for the area so that they describe a perfect square. Then, write the two equivalent equations for the area of the square.

a. $x^2 + 4x + \underline{4}$

b. $x^2 + 6x + \underline{9}$

c. $x^2 + \frac{8x}{2} + \underline{16}$

d. $x^2 + 12x + 36$

$\left(\frac{b}{2}\right)^2$

7. If $x^2 + bx + c$ is a perfect square, what is the relationship between b and c? How do you use b to find c, like in problem 6?

$$\left(\frac{b}{2}\right)^2 = c$$

$$\sqrt{\left(\frac{b}{2}\right)^2} = \sqrt{c}$$

$$2 \cdot \frac{b}{2} = \sqrt{c} \cdot 2$$

$$b = \sqrt{c} \cdot 2$$

Will this strategy work if b is negative? Why or why not?

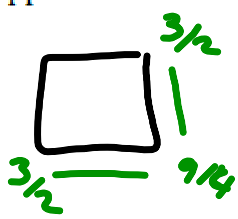
$$x^2 - 8x + \underline{16}$$

yes, the negative is squared.

Will the strategy work if b is an odd number? What happens to c if b is odd?

$$x^2 + 3x + \underline{9/4}$$

$$\left(\frac{3}{2}\right)^2 = \frac{9}{4}$$



STRUCTURES OF EXPRESSIONS – 2.3

2.3

READY, SET, GO! Name Period Date

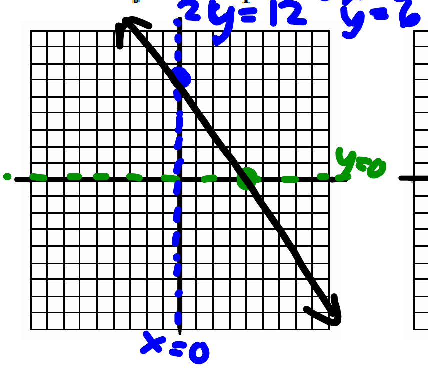
READY

Topic: Graphing lines using the intercepts

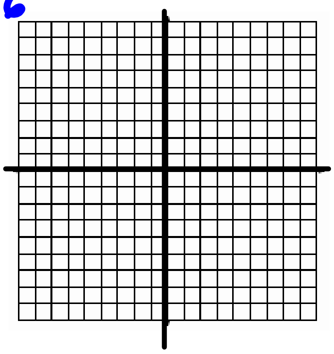
Find the x-intercept and the y-intercept. Then graph the equation.

$0+2y=12$
$3x+0=12$

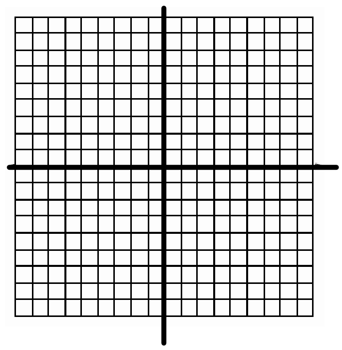
1. $3x+2y=12$
   a. x-intercept: sub $y=0$
   $3x=12$ $x=4$
   b. y-intercept: sub $x=0$
   $2y=12$ $y=6$



2. $8x-12y=-24$
   a. x-intercept:
   b. y-intercept:



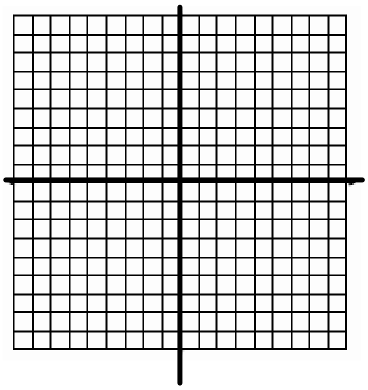
3. $3x-7y=21$
   a. x-intercept:
   b. y-intercept:



4. $5x - 10y = 20$

a. x-intercept:

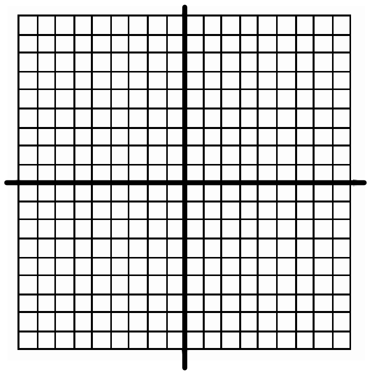
b. y-intercept:



5. $2y = 6x - 18$

a. x-intercept:

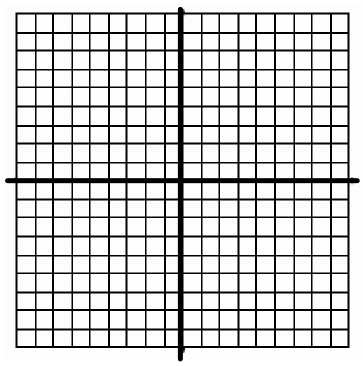
b. y-intercept:



6. $y = -6x + 6$

a. x-intercept:

b. y-intercept:



SET

Topic: Completing the square by paying attention to the parts

Multiply. Show each step. Circle the pair of <u>like terms</u> before you simplify to a trinomial.

7. $(x + 5)(x + 5)$ 8. $(3x + 7)(3x + 7)$ 9. $(9x + 1)^2$ 10. $(4x + 11)^2$

11. Write a rule for finding the coefficient "B" of the x-term (the middle term) when multiplying and simplifying $(ax + q)^2$.

In problems 12 – 17, (a) Fill in the number that completes the square.

$\sqrt{16} \cdot 2 = 4 \cdot 2 = 8$ **(b) Then write the trinomial as the product of two factors.** ( )( )

12. a) $x^2 + 8x +$ 16 $\div 2$

b) $(x+4)(x+4)$

13. a) $x^2 + 10x +$ 25 $\div 2$

b) $(x+5)(x+5)$

14. a) $x^2 + 16x +$ ____

b)

15. a) $x^2 + 6x +$ ____

b)

16. a) $x^2 + 22x +$ ____

b)

17. a) $x^2 + 18x +$ ____

b)

In problems 18 – 26, (a) Find the value of "B," that will make a perfect square trinomial.
(b) Then write the trinomial as a product of two factors.

$\sqrt{16}\cdot 2 = 4\cdot 2 = 8$ $\sqrt{121}\cdot 2 = 11\cdot 2 = 22$

18. $x^2 + Bx + 16$
    a) 8 → 1/2
    b) $(x+4)(x+4)$

19. $x^2 + Bx + 121$
    a) 22
    b) $(x+11)(x+11)$

20. $x^2 + Bx + 625$
    a)
    b)

21. $x^2 + Bx + 225$
    a)
    b)

22. $x^2 + Bx + 49$
    a)
    b)

23. $x^2 + Bx + 169$
    a)
    b)

24. $x^2 + Bx + \frac{25}{4}$
    a)
    b)

25. $x^2 + Bx + \frac{9}{4}$ $\sqrt{\frac{9}{4}} = \left(\frac{3}{2}\right)\cancel{2} = 3$
    a) 3 → 1/2
    b) $\left(x+\frac{3}{2}\right)\left(x+\frac{3}{2}\right)$

26. $x^2 + Bx + \frac{49}{4}$ $\sqrt{\frac{49}{4}}\cdot 2 = \frac{7}{\cancel{2}}\cancel{2}$
    a) 7
    b) $\left(x+\frac{7}{2}\right)\left(x+\frac{7}{2}\right)$

GO

Topic: Features of horizontal and vertical lines

Find the intercepts of the graph of each equation. State whether it's an x- or y- intercept.

27. y = -4.5

y-intercept

28. x = 9.5

29. x = -8.2

30. y = 112