

3.6 Remainder Theorem Worksheet

Period\_

Is the given a divisor a Factor? Yes or No. Show long division.

1)  $x^3 - 9x^2 + 24x + 20 = 0; x + 2$       2)  $x^3 + 3x^2 - 24x - 80 = 0; x - 5$

Handwritten work for problem 1:

$$\begin{array}{r}
 x^2 + 7x + 10 \\
 x+2 \overline{) x^3 - 9x^2 + 24x + 20} \\
 \underline{-x^3 - 2x^2} \phantom{+ 20} \\
 7x^2 + 24x + 20 \\
 \underline{-7x^2 - 14x} \phantom{+ 20} \\
 10x + 20 \\
 \underline{-10x - 20} \\
 0
 \end{array}$$

$(x+2)(x+5)(x+2)$

yes it is a factor.



3)  $(r^3 + 5r^2 + 15r + 25) \div (r + 2)$

$$\begin{array}{r}
 r^2 + 3r + 9 \\
 r+2 \overline{) r^3 + 5r^2 + 15r + 25} \\
 \underline{-r^3 - 2r^2} \phantom{+ 25} \\
 3r^2 + 15r + 25 \\
 \underline{-3r^2 - 6r} \phantom{+ 25} \\
 9r + 25 \\
 \underline{-9r - 18} \\
 0 \neq 7R
 \end{array}$$

NOT a factor

4)  $x^4 + 9x^3 + 26x^2 + 34x + 20 = 0$        $(x+1-i)$        $(x+1+i)$

	$x+1-i$	
$x$	$x^3$	$x-x-i$
$+1$	$x$	$-i$
$+i$	$x$	$-i$

$x^2 + 2x + 2$

$$\begin{array}{r}
 x^2 + 7x + 10 \\
 x+2 \overline{) x^4 + 9x^3 + 26x^2 + 34x + 20} \\
 \underline{-x^3 - 2x^2} \phantom{+ 34x + 20} \\
 7x^2 + 24x + 20 \\
 \underline{-7x^2 - 14x} \phantom{+ 20} \\
 10x + 20 \\
 \underline{-10x - 20} \\
 0
 \end{array}$$

5)  $x^4 - 2x^3 - 8x^2 - 30x - 25 = 0; (x+1-2i)$   
 $(x+1+2i)$

	$x$	$x^2$	$x$	$-2i$
$x$	$x^2$	$x$	$-2i$	
$+1$	$2x$	$-1$	$-2i$	
$+2i$	$2x$	$2i$	$-4i$	

$x^2 + 2x + 5$

$-4(-1) = 4$

$x^2 - 4x - 5$

$$\begin{array}{r} x^2 + 2x + 5 \overline{) x^4 - 2x^3 - 8x^2 - 30x - 25} \\ \underline{-x^2 - 2x - 5} \phantom{-25} \\ 4x^2 - 30x - 25 \\ \underline{-4x^2 + 16x + 20} \\ 13x^2 - 30x - 25 \\ \underline{-13x^2 + 52x + 65} \\ 22x - 40 \\ \underline{-22x + 44} \\ 4 \end{array}$$

6)  $x^4 - 66x^2 - 256x - 255 = 0; (x-4-\sqrt{33})$   
 $(x-4+\sqrt{33})$

	$x$	$x^2$	$x$	$-\sqrt{33}$
$x$	$x^2$	$x$	$-\sqrt{33}$	
$-4$	$-4x$	$16$	$4\sqrt{33}$	
$\sqrt{33}$	$\sqrt{33}x$	$-33$		

$x^2 - 8x - 17$

$-\sqrt{33} \cdot \sqrt{33}$   
 $-(\sqrt{33})^2 = -33$

$x^2 - 8x - 17 \overline{) x^4 + 0x^3 - 66x^2 - 256x - 255}$

Find all roots.

7)  $x^4 - 25 = 0$

4 answers!

$$(x^2)^2 - 25$$

$$\frac{-25}{5 \times -5}$$

$$(x^2 + 5)(x^2 - 5)$$

$$x^2 + 5 = 0$$

$$x^2 - 5 = 0$$

$$\sqrt{x^2} = \sqrt{-5}$$

$$\sqrt{x^2} = \sqrt{5}$$

$$x = \pm\sqrt{5}i \quad x = \pm\sqrt{5}$$

9)  $x^4 - x^2 - 72 = 0$

$$\frac{-72}{-9 \times 8}$$

$$(x^2 - 9)(x^2 + 8)$$

$$x^2 - 9 = 0$$

$$x^2 + 8 = 0$$

$$\sqrt{x^2} = \sqrt{9}$$

$$\sqrt{x^2} = \sqrt{-8}$$

$$x = \pm 3$$

$$x = \pm\sqrt{8}i$$

8)  $x^3 + 2x^2 - 5x - 10 = 0$

$\pm 2, 5, 1, 10$

Test 2:  $2^3 + 2(2)^2 - 5(2) - 10 = 0?$   
 $8 + 8 - 10 - 10 = 0$  X

Test -2:  $(-2)^3 + 2(-2)^2 - 5(-2) - 10 = 0?$   
 $-8 + 8 + 10 - 10 = 0$  V

$$(x+2) \overline{) x^3 + 2x^2 - 5x - 10}$$

$$x = \pm\sqrt{5}$$

10)  $x^3 + 9x^2 + 20x = 0$

$$x(x^2 + 9x + 20)$$

$$\frac{20}{9}$$

11) Solve for x:  $x^2 - 6x + 2 = 0$

*Handwritten notes: a, b, c; CANT; -b*

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-6) \pm \sqrt{(-6)^2 - 4(1)(2)}}{2(1)}$$

$$= \frac{6 \pm \sqrt{36 - 8}}{2}$$

$$= \frac{6 \pm \sqrt{28}}{2}$$

*Handwritten notes: 4, 2, 2*

$$= \frac{6 \pm 2\sqrt{7}}{2}$$

$$x = 3 \pm \sqrt{7} \quad (x - 3 + \sqrt{7})(x - 3 - \sqrt{7})$$

**Remainder Theorem**

12) How can you predict whether a factor will divide evenly or not?

Test your prediction:

I. List the possible factors of

$f(x) = x^3 - 6x^2 + 12x - 8$

*Handwritten notes: ±2, ±4, ±8*

II. Test 3 of the possible factors.

a.  $f(1) =$

b.  $f(2) =$

c.  $f(4) =$

III. Test 1 impossible factor.

d.  $f(5) =$

Evaluate each function at the given value AND state whether or not it is a zero.

13)  $f(a) = 2a^3 - 10a^2 + 14a - 17$  at  $a = 3$  ? *NO!*

*Handwritten notes: 2(3)^3 - 10(3)^2 + 14(3) - 17 = 0? NO! 54 - 90 + 42 - 17 = -11 ≠ 0*

14)  $f(x) = 5x^3 + 14x^2 - 6x$  at  $x = -3$

15)  $f(a) = a^3 - 3a^2 - 18a$  at  $a = -3$

16)  $f(m) = m^3 + 2m^2 - 13m - 20$  at  $m = -4$

17)  $f(n) = n^3 + n^2 - 12n + 6$  at  $n = 3$

Write a polynomial function of least degree that has real coefficients, the following zeros, and a leading coefficient of 1.

18)  $-1, -2i, 2i$

19)  $3, 5, -4, -1$

20)  $-3, 3 + \sqrt{7}, 3 - \sqrt{7}$   
 $(x+3)(x-3+\sqrt{7})(x-3-\sqrt{7})$

21)  $-5, -2, -2+3i, -2-3i$   
 $(x+5)(x+2)(x+2+3i)(x+2-3i)$

$x^2+2x+5x+10$

$(x^2+7x+10)$

standard form

$x^4+11x^3+51x^2+131x+130$

$(x^2+4x+13)$

$x^2$	$x^4$	$4x^3$	$13x^2$
$7x$	$7x^3$	$28x^2$	$91x$
$10$	$10x^2$	$40x$	$130$

Perform the indicated operation.

$$22) \begin{aligned} g(x) &= 2x + 4 \\ h(x) &= 2x^3 - 3x \\ \text{Find } g(x) - h(x) \end{aligned}$$

$$(2x + 4) - (2x^3 - 3x)$$

$$24) \begin{aligned} g(t) &= t^3 + t \\ h(t) &= t - 1 \\ \text{Find } g(t) - h(t) \end{aligned}$$

$$26) \begin{aligned} h(x) &= x^2 + 3 \\ g(x) &= 4x - 1 \\ \text{Find } h(x) + g(x) \end{aligned}$$

$$23) \begin{aligned} g(x) &= 3x + 5 \\ h(x) &= x + 4 \\ \text{Find } g(x) - h(x) \end{aligned}$$

$$(3x + 5) - (x + 4)$$

$$\begin{array}{r} 3x + 5 - x - 4 \\ \hline 2x + 1 \end{array}$$

$$25) \begin{aligned} h(t) &= -t^3 - 4t \\ g(t) &= 3t - 2 \\ \text{Find } h(t) \cdot g(t) \end{aligned}$$

$$27) \begin{aligned} f(x) &= -3x^3 + 5 \\ g(x) &= 3x + 1 \\ \text{Find } f(x) \cdot g(x) \end{aligned}$$