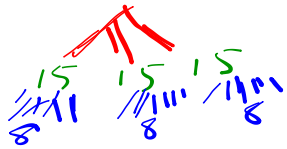
FUNDAMENTAL COUNTING PRINCIPLE

If decision M can be made m ways and decision N can be made n ways, then the two decisions can be made $m \cdot n$ ways.

EXAMPLES



1. The ice cream shop offers a choice of a 3 cone sizes, 15 flavors, and 8 toppings. How many cones are possible if you can only choose one flavor and one topping?

$3 \cdot 15 \cdot 8 = 360$

2. Virginia license plates consist of three letters followed by 4 digits. How many different license plates are possible?

$\underbrace{26 \cdot 26 \cdot 26}_{\text{3 letters}} \cdot \underbrace{10 \cdot 10 \cdot 10 \cdot 10}_{\text{4 digits}} = 26^3 \cdot 10^4 = 175{,}760{,}000$

3. Nick's science test has a section with 12 true or false questions. How many different ways can Nick answer these questions?

$\underbrace{2 \cdot 2 \cdot 2 \cdots 2}_{12} = 2^{12} = 4096$

locker

$50 \cdot 50 \cdot 50 = 125{,}000$

4. To enter their home, the Clayton family enters a 4-digit code. How many codes are possible?

$10 \cdot 10 \cdot 10 \cdot 10 = 10^4 = 10{,}000$

FACTORIALS

$n!$

The product of all natural numbers from n to 1.

Examples:

$9! = 9 \cdot 8 \cdot 7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1 = 362{,}880$

$12! = 479{,}001{,}600$ $8! = 40{,}320$ $\frac{17!}{11!} = \frac{17 \cdot 16 \cdot 15 \cdot 14 \cdot 13 \cdot 12 \cdot 11 \cdots}{11 \cdot 10 \cdot 9 \cdot 8 \cdots} = 89{,}910{,}720$

*$0! = 1$

PERMUTATIONS

${}_nP_r$

An **arrangement** or **line up** of objects in which order matters.

Permutation Formula: $${}_nP_r = \frac{n!}{(n-r)!}$$

n = total number of objects available, options

r = number of objects to use for the arrangement use

***Important Shortcut:** $${}_nP_n = n!$$

EXAMPLES

5. How many ways can you arrange the letters in the word **DINOSAUR**?

8 letters

$${}_8P_8 = \frac{8!}{(8-8)!} = \frac{8!}{0!} = \frac{8!}{1} = 8! = 40{,}320$$

6. Seven students are competing in a geography bee. How many ways can they finish in first, second, and third place?

$7 \cdot 6 \cdot 5 = 210$

$${}_7P_3 = \frac{7!}{(7-3)!} = \frac{7!}{4!} = \frac{7 \cdot 6 \cdot 5 \cdot 4 \cdot 3 \cdot 2 \cdot 1}{4 \cdot 3 \cdot 2 \cdot 1}$$ ✓

7. Melanie is taking four classes this semester: American History, Algebra 2, AP English, and Chemistry. How many ways can these four classes be arranged on her schedule?

$_4P_4 = 4! = 4 \cdot 3 \cdot 2 \cdot 1 = 24$

8. There are 16 players on the baseball team. How many ways can the coach make a 9-player batting order?

$_{16}P_9 \quad \frac{16!}{7!} = 4,151,472,00$

9. There are 28 students in Mr. Miller's homeroom. How many ways can the students elect a student council representative and alternate?

$_{28}P_2 \quad \frac{28!}{26!} = 28 \cdot 27 = 756$

PERMUTATIONS

With Repetition

10. How many different 10-letter arrangements are possible using the letters in the word **AUTOMOBILE**?

$\frac{10!}{2!}$ options / repeats $= 1814400$

11. How many different 9-letter arrangements are possible using the letters in the word **DISAPPEAR**?

$\frac{9!}{(2!2!)} = 90,720$

COMBINATIONS

$_nC_r$

A **group** of objects in which order doesn't matter!

Combination Formula: $_nC_r = \frac{n!}{r!(n-r)!}$

n = total number of objects available, options

r = number of objects to use for the arrangement choose

***Important Shortcut:** $_nC_n = 1$

EXAMPLES

12. Natalie has 16 close friends. How many ways can she choose 5 to be bridesmaids in her wedding?

$_{16}C_5 = \frac{16!}{5!(16-5)!} = \frac{16!}{5! \cdot 11!} = 4368$

13. There are 24 students in Kyle's kindergarten class. How many ways can he choose eight to attend his birthday party?

$_{24}C_8 = \frac{24!}{8!(24-8)!} = \frac{24!}{(8!16!)} =$

14. Abby is adopting kittens from the pet store. If there are 18 kittens, how many ways can she choose two?

15. There are twelve employees at the sub shop. How many ways can the manager choose four for the Sunday evening shift?

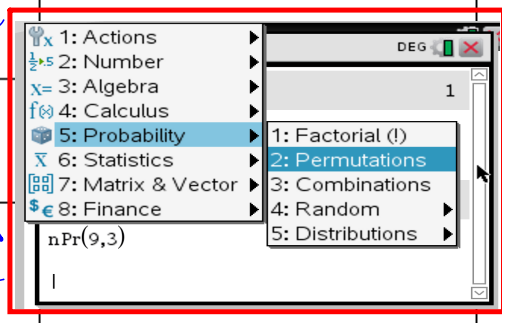
16. There are 85 players on the football team. How many ways can the coach choose three to represent the team in the coin toss?

Permutation or Combination?

Directions: Recall that **a permutation is an arrangement** with a specific order, while a **combination is a group** with no specific order. Determine whether the example represents a permutation or combination, then solve.

order ← no order

	Example	P or C?	Answer
1	How many ways can 4 candy bars be chosen from a store that sells 30 candy bars?	C	${}_{30}C_4$ $\frac{30!}{4!26!}$ (27,405)
2	How many ways can 13 students line up for lunch? order	P	${}_{13}P_{13} = 13!$ 6,227,020,800
3	How many ways can you make a 3-letter arrangements out of the letters in the word **TRAPEZOID**. order	P	${}_9P_3 = \frac{9!}{6!} = 504$
4	How many ways can you choose 2 books from a shelf of 40 books?	C	
5	How many ways can 12 swimmers finish in first, second, and third place?	P	
6	How many ways can Mrs. Sullivan choose two students from 27 to help put away calculators at the end of class?	C	
7	You have enough tickets to play 6 different games at the amusement park. If there are 14 games, how many ways can you choose six?	C	
8	How many different ways can 9 trumpet players in the marching band line up?	P	
9	Seven students worked together on a project. How many ways can their teacher choose four to present the project?	C	
10	There are 18 offensive players on the hockey team. How many ways can the coach choose a left wing, center, and right wing to start the game?	P	
11	How many different 12-letter arrangements can be made using the letters in the word **INDIANAPOLIS**?	P	$\frac{12!}{(3!\cdot 2!\cdot 2!)}$
12	There are 26 gold fish in the tank at the store. How many ways can Ben choose five?	C	



Name: _____ **Unit 11:** Probability & Statistics

Date: _____ **Bell:** _____ **Homework 1:** Fundamental Counting Principle, Permutations & Combinations

**** This is a 2-page document! ****

Directions: Find the total number of outcomes that are possible.

1. In a school building, there are 8 exterior doors and 12 stairways to enter the second floor. Find the total number of ways to reach the second floor.

2. Kendall went shopping and purchased 7 shirts, 3 pairs of pants, 2 jackets. How many different outfits are possible?

$7 \cdot 3 \cdot 2 = \boxed{42}$

3. An internet passcode consists of a digit followed by a letter, followed by another digit. Assuming the digits are 0-9. How many different passcodes are possible?

4. Grant is rolling a standard six-sided number die eight times. How many outcomes are possible?

$6^8 = \boxed{1,679,616}$

5. A quiz has eight multiple choice questions with four options for each (A, B, C, and D). How many ways are there to answer the questions?

6. When Jack bought his new truck, there were 96 different ways his truck could be equipped. He had four choices of engines and two choices of transmissions. If the only other choice was color, how colors were available?

$4 \cdot 2 \cdot x = 96$

$8x = 96$ $\boxed{x = 12}$

Directions: Evaluate the following.

7. $6!$

8. $\frac{16!}{7!}$

9. $\frac{7!}{10!}$

0.00138

10. $\frac{2! \cdot 9!}{(10-7)!}$

$\frac{725760}{6} = 120960$

11. ${}_{20}P_4$

12. ${}_{15}C_8$

13. ${}_{11}P_7$

$98,017,920$

14. ${}_{27}C_{22}$

$80,730$

15. Find ${}_9P_9$ and ${}_9C_9$. Why do the answers differ? Explain.

_____

Directions: Determine whether the situations represent permutations or combinations. Then solve.

Example	P/C	Solve
16. Scott has 7 chores to complete this Saturday. How many ways can he arrange the order in which he does them?		
17. There are 16 different colored markers in the bucket. How many ways can Kelly choose five of them?	C	$_{16}C_5$ $\frac{16!}{(5!\cdot 11!)}$
18. How many ways are there to elect a President, Vice President, Secretary, and Treasurer, from a club with 32 members?		
19. The dance company is choosing 3 new dancers from a group of 25 who try out. How many ways can they choose the new dancers?	C	
20. There are ten toppings available to make an ice cream sundae. How many ways can Max choose two?		
21. How many different 11-letter arrangements can be made using the letters in the word **FIRECRACKER**?	P*	$\frac{11!}{3!2!2!}=$
22. Mindy is purchasing songs from an album on iTunes. If the album has 14 songs, how many ways can she choose 6?		
23. Carl is choosing a four-character password for his cell phone. The password can contain letters and digits only, and can't include repeating characters. How many passwords are possible? $26+10=36$ $36\cdot35\cdot34\cdot33$	P	$_{36}P_4$
24. Mr. Pratt has 26 students in his math class. He has three prizes to give away: a pencil, an eraser, and a no homework coupon. How many ways can he choose three students to win these awards?		
25. How many ways are there to choose seven cards from a deck of 52?	C	$_{52}C_7$

Main Ideas/Questions	Notes/Examples
Theoretical Probability	• Probability is the measure of how likely an event is to occur. • The set of all possible outcomes is called the **sample space** • For equally likely outcomes, the theoretical probability of an event, $P(E)$, is the ratio of the number of favorable outcomes to the total number of outcomes possible.

Simple Events

The probability of one event.

prime
2, 3, 5, 7, 11
13, 17, 19, 23,
29, 31

4.5 · 6,4
3,6
6,6
6,5
5,5

1. A jar contains 32 red marbles and 28 blue marbles. What is the probability that a randomly selected marble is red?

total = 60

$\frac{32 \text{ red}}{60 \text{ total}} = \frac{8}{15}$

2. A letter in the word **RESTORATION** is randomly selected. What is the probability of selecting a vowel?

$\frac{5 \text{ vowels}}{11 \text{ letters}}$ $\left(\frac{5}{11}\right)$

3. A day in the month of January is randomly selected. What is the probability of selecting a prime number?

31 days

$\frac{11 \text{ primes}}{31 \text{ days}} = \frac{11}{31}$

4. Two dice are rolled. What is the probability that the sum of the two dice on the next roll is at least 9?

4.5 $\frac{10}{36} = \left(\frac{5}{13}\right)$

5. What is the probability of drawing a heart or a club from a standard deck of cards?

52
black (26) red (26)
13 13
13 13

$\frac{26}{52} = \left(\frac{1}{2}\right)$

6. There are 8 books lettered A-H on the shelf. If Scott randomly chose two books, what is the probability that he chose books A and B?

Complement of an Event

Opposite

The complement of an event is the probability of the event Not happening. Since the sum of all probabilities in sample space is 100% 1, the probability of an event not happening is $P(\sim E) =$ $1 - P(E)$.

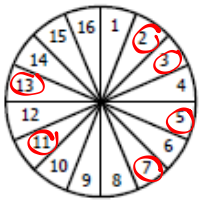
7. The probability that it will snow tomorrow is 7/20. What is the probability that it will not snow?

$1 - \frac{7}{20} = \left(\frac{13}{20}\right)$

8. A month of the year is randomly selected. What is the probability of getting a month that does not begin with the letter A?

April, May

$1 - \frac{2}{12} = \left(\frac{10}{12}\right)$



9. If the spinner to the left is spun, find the probability that it lands on a number that is not prime.

$1 - \frac{6}{16} = \frac{10}{16} = \left(\frac{5}{8}\right)$

10. Two dice are rolled. What is the probability of not getting doubles?

$1 - \frac{6}{36} = \frac{30}{36} = \left(\frac{5}{6}\right)$ $\frac{6}{36}$

Rolling Dice

	1	2	3	4	5	6
1	(1,1)	(1,2)	(1,3)	(1,4)	(1,5)	(1,6)
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)	(2,6)
3	(3,1)	(3,2)	(3,3)	(3,4)	(3,5)	(3,6)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)	(4,6)
5	(5,1)	(5,2)	(5,3)	(5,4)	(5,5)	(5,6)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)	(6,6)

Compound Events

The probability of two or more simple events.

Independent Events

When the outcome of one event does not affect the outcome of the other event.

$P(A \text{ and } B) =$

$P(A) \cdot P(B)$

Dependent Events

When the outcome of one event does affect the outcome of the other event.

$P(A \text{ and } B) =$

$P(A) \cdot P(B \mid \text{after A happened})$

Independent Events

11. A die is rolled 3 times. What is the probability of getting 1's on each roll?

$\frac{1}{6} \cdot \frac{1}{6} \cdot \frac{1}{6} = \frac{1}{216}$

12. A coin is tossed, then a day of the week is selected. What is the probability of getting tails then a day starting with the letter T?

$\frac{1}{2} \cdot \frac{2}{7} = \frac{1}{7}$ Tues Thurs

A bag contains 8 red crayons, 14 purple crayons, 6 yellow crayons, and 4 green crayons. A crayon is selected, replaced, then another is selected. Find each probability. total = 32

13. P(purple then yellow)

$\frac{14}{32} \cdot \frac{6}{32} = \frac{(14 \cdot 6)}{(32 \cdot 32)} = \frac{21}{256}$

14. P(green then red)

15. P(two purples)

$\frac{14}{32} \cdot \frac{14}{32} = \frac{49}{256}$

16. P(two yellows)

Dependent Events

~~Using the same~~ example from above, assume once a crayon is selected, it is ~~NOT replaced~~. Find each probability.

17. P(yellow then red)

$\frac{6}{32} \cdot \frac{8}{31} = \frac{3}{62}$

18. P(purple then green)

19. P(two reds)

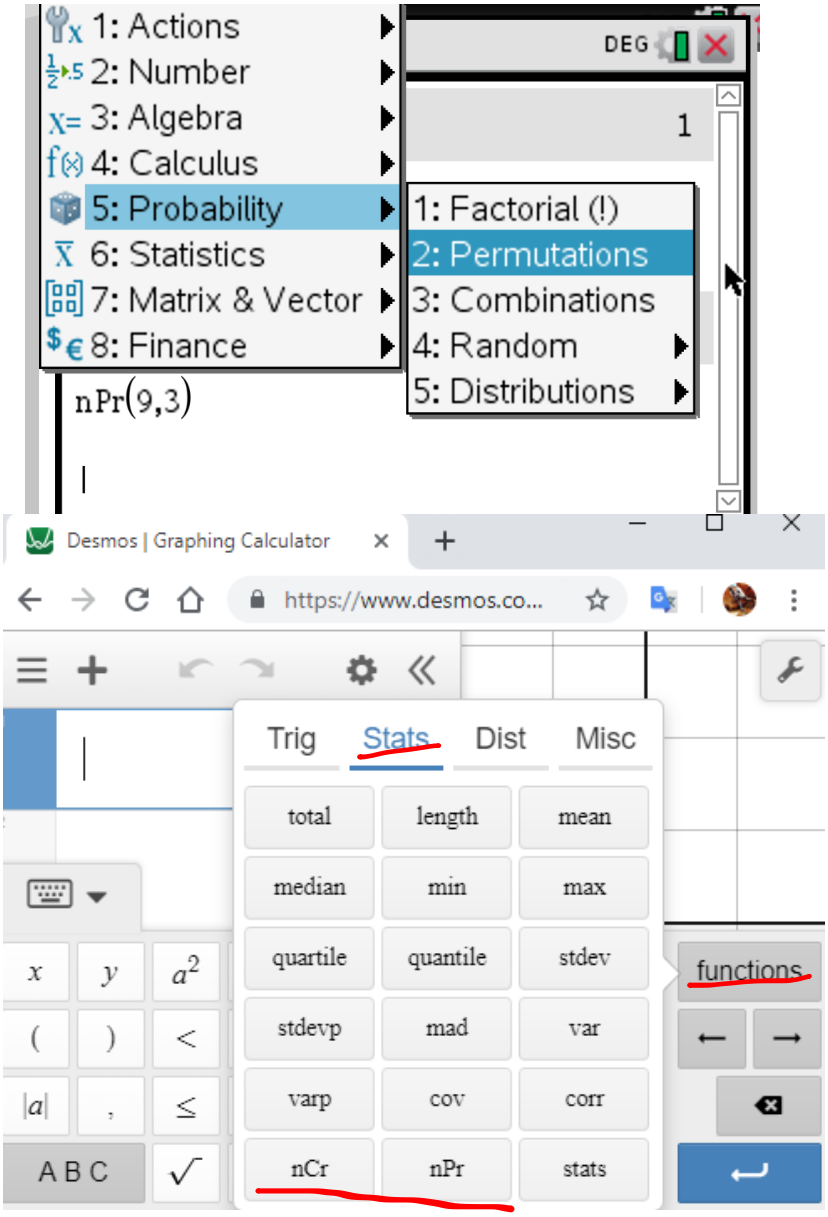
$\frac{8}{32} \cdot \frac{7}{31} = \frac{7}{124}$

20. P(two greens)

21. A card is drawn from a standard deck, not replaced, and another is drawn. What is the probability of choosing a heart then a spade?

$\frac{13}{52} \cdot \frac{13}{51}$

22. Jack had four Snicker bars and 8 Mars bars. He randomly chose a piece of candy, ate it, then chose another. What is the probability that both candy bars were Snickers?


1: Actions
2: Number
3: Algebra
4: Calculus
5: Probability
6: Statistics
7: Matrix & Vector
8: Finance
DEG
1
1: Factorial (!)
2: Permutations
3: Combinations
4: Random
5: Distributions
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Name: _____ **Unit 11:** Probability & Statistics

Date: _____ **Bell:** _____ **Homework 2:** Theoretical Probability

Odds

**** This is a 2-page document! ****

Part I: Simple Probability

Use for questions 1-3: A random two-digit number (10-99) is drawn. Find each probability.

1. $P(32)$

2. $P(\text{odd number})$

3. $P(\text{a multiple of 5})$

Use for questions 4-6: A letter is randomly chosen from the word CANDLESTICK. Find each probability.

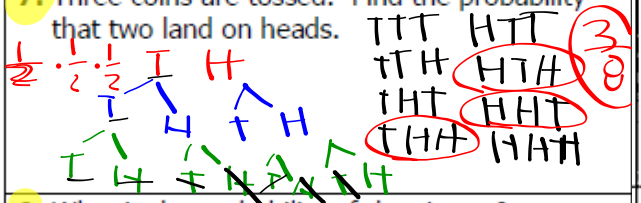
4. $P(\text{a vowel})$

5. $P(N \text{ or } S)$

6. $P(\text{not } C)$

$1 - \frac{2}{11} = \boxed{\frac{9}{11}}$

7. Three coins are tossed. Find the probability that two land on heads.



8. A month is randomly chosen. What is the probability that the month chosen has less than 31 days?

$1 - \frac{7}{12} = \boxed{\frac{5}{12}}$

9. What is the probability of drawing a 9 or diamond from a standard deck of cards?

10. Credit cards place a three-digit security code on the back of cards. What is the probability that a code starts with the number 7?

$\frac{1 \cdot 10 \cdot 10}{10 \cdot 10 \cdot 10} = \boxed{\frac{1}{10}}$

11. Two dice are rolled. What is the probability of not getting doubles?

12. Mikayla has the following songs on her iPod: 14 Taylor Swift songs, 16 Meghan Trainor songs, and 17 Katy Perry songs. What is the probability that the next song that plays is not Katy Perry?

$1 - \frac{17}{47} = \boxed{\frac{30}{47}}$

Part II: Compound Probability

13. A dice is rolled, then a coin is tossed. What is the probability of getting a 5 then tails?

14. A coin is tossed, then a number 1-10 is chosen at random. What is the probability of getting heads then a number less than 4?

$\frac{1}{2} \cdot \frac{3}{10} = \boxed{\frac{3}{20}}$

15. Natalie guessed on the last four true or false questions on her math quiz. What is the probability that she got all four questions correct?

16. A card is drawn from a standard deck and a letter is chosen from the word **INCREDIBLE.** What is the probability of drawing a king then getting an I?

$\frac{4}{52} \cdot \frac{2}{10} = \boxed{\frac{1}{65}}$

Use for questions 17-20: A bag contains 30 lottery balls numbered 1-30. A ball is selected, replaced, then another is drawn. Find each probability.

17. *P*(and even, then odd)

18. *P*(7, then a number greater than 16)

$\frac{1}{30} \cdot \frac{14}{30} = \boxed{\frac{7}{450}}$

19. *P*(a multiple of 5, then a prime number)

20. *P*(two even numbers)

$\frac{15}{30} \cdot \frac{15}{30} = \boxed{\frac{1}{4}}$

Use for questions 21-24: A bag contains 30 lottery balls numbered 1-30. A ball is selected, NOT replaced, then another is drawn. Find each probability.

21. *P*(a 2-digit number, then 4)

↳ 10 digiger

$\frac{21}{30} \cdot \frac{1}{29} = \frac{7}{290}$

take one out

22. *P*(19, then a multiple of 4)

$\frac{1}{30} \cdot \frac{7}{29} = \boxed{\frac{7}{870}}$

23. *P*(24, then a number less than 15)

24. *P*(two perfect squares)

$\frac{5}{30} \cdot \frac{4}{29} = \boxed{\frac{2}{87}}$

25. A football team has 5 freshmen, 8 sophomores, 11 juniors, and 16 seniors. If two are chosen at random to participate in the coin toss, what is the probability that both players chosen are seniors?

total: 40

$\frac{16}{40} \cdot \frac{15}{39} = \frac{2}{13}$

26. Ryan's mom randomly chooses two days each week for Ryan to do his chores. What is the probability that she picks Saturday and Sunday?