PASCAL'S TRIANGLE

Pascal's triangle was a pattern of numbers that was discovered in the 13th century. Each number in Pascal's triangle is the sum of the two numbers diagonally above it. All outside numbers are 1.

Complete rows 4 and 5 of Pascal's triangle below:

Row 0 → x^0 →

Row 1 → $a^1 + b^1$

Row 2 → $1a^2 + 2ab + 1b^2$

Row 3 →

Row 4 →

Row 5 →

row 6

						1					
					1		1				
				1		2		1			
			1		3		3		1		
		1		4		6		4		1	
	1		5		10		10		5		1
1		6		15		20		15		6		1

Expanding Binomials

Expand the binomial $(a + b)^3$:

$a^2 + 2ab + b^2$

a	a^3	$2a^2b$	ab^2
b	a^2b	$2ab^2$	b^3

$(a+b)(a+b)(a+b)$

$a^2 + ab + ab + b^2$ → row 2

$(a^2 + 2ab + 1b^2)(a+b)$

$a^3 + 3a^2b + 3ab^2 + 1b^3$ → row 3

What do you notice about the coefficients?

THE BINOMIAL THEOREM

If n is a natural number, then $(a + b)^n =$

$$\cancel{{}_nC_0 \cdot a^n b^0} + {}_nC_1 \cancel{a^{n-1}} b^1 + {}_nC_2 \cancel{a^{n-2}} b^2 + \ldots + {}_nC_n \cdot a^0 b^n = \boxed{\sum_{k=0}^{n} {}_nC_k \cdot a^{n-k} b^k}$$

Examples

Directions: Use the binomial theorem to expand each binomial.

1. $(a+b)^5$

$1a^5b^0 + 5a^4b + 10a^3b^2 + 10a^2b^3 + 5a^1b^4 + 1a^0b^5$

${}_5C_0$ ${}_5C_1$ ${}_5C_2$ ${}_5C_3$ ${}_5C_4$ ${}_5C_5$

2. $(x + y)^7$

${}_7C_0$ ${}_7C_1$ ${}_7C_2$ ${}_7C_3$ ${}_7C_4$ ${}_7C_5$ ${}_7C_6$ ${}_7C_7$

$1x^7 + 7x^6y + 21x^5y^2 + 35x^4y^3 + 35x^3y^4 + 21x^2y^5 + 7xy^6 + 1y^7$

	3. $(c+d)^{10}$
Coefficients Other than 1	**4.** $(x-3)^6$ ✓ $1x^6 \quad 6x^5(-3)^1 \quad 15x^4(-3)^2 \quad 20x^3(-3)^3 \quad 15x^2(-3)^4 \quad 6x(-3)^5 \quad 1(-3)^6$ $1x^6 - 18x^5 + 135x^4 - 540x^3 + 1215x^2 - 1458x + 729$
	5. $(2m+n)^7$ $1 2^7 m^7 \quad 7 2^6 m^6 n \quad 21 2^5 m^5 n^2 \quad 35 2^4 m^4 n^3 \quad 35 2^3 m^3 n^4 \quad 21 2^2 m^2 n^5 \quad 7 2 m n^6 \quad 1 n^7$ $128m^7 + 448m^6n + 672m^5n^2 + 560m^4n^3 + 280m^3n^4 + 84m^2n^5 + 14mn^6 + 1n^7$
	6. $(k+2)^8$
	7. $(3p-2q)^5$

Observations

In the binomial expansion of $(a+b)^n$: $a^n \rightarrow \leftarrow b^n$

- The total number of terms is always $n+1$.
- The exponent of a in the first term is n.
- The exponent of b in the last term is n.
- The exponent of a decrease by 1 from left to right.
- The exponent of b increase by 1 from left to right.
- The sum of the exponents in each term is n.
- The coefficients are symmetrical and follow the nth row of pascal's $\triangle$.

Binomial Experiments

A **binomial experiment** consists of n **independent trials** in which:

- There are only **two outcomes**: success and failure
- The probability of success p is the same in every trial.
- The probability of failure q is the same in every trial.
- The probability of success is p.
- The probabiltity of failure is $q = (1 - p)$

Examples

$\left(1 - \frac{1}{12}\right)$

Experiment	Success	Failure	p	q
Randomly selecting a month 30 times	January	Not January	$\frac{1}{12}$	$\frac{11}{12}$
Randomly selecting a card 100 times	Heart $\frac{13}{52}$	Not a Heart	$\frac{1}{4}$	$\frac{3}{4}$

Binomial Probability

If a binomial experiment has n trials in which p is the probability of success and q is the probability of failure, then the binomial probability that there will be exactly r successes is:

$$\boxed{{}_nC_r \cdot p^r \cdot q^{n-r}}$$

Examples

(with an exact number of successes)

1. Mark tosses a coin 25 times. What is the probability that it lands on heads **exactly** seven times?

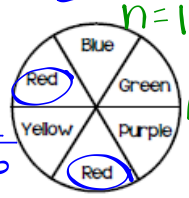
$n = 25 \quad r = 7 \quad p = \frac{1}{2} \quad q = \frac{1}{2}$

${}_{25}C_7 \cdot \left(\frac{1}{2}\right)^7 \cdot \left(\frac{1}{2}\right)^{25-7 = 18}$

$= .0143$

1.43%

2. The spinner below is spun 10 times. What is the probability that it lands on red **exactly** six times?



$n = 10 \quad r = 6 \quad p = \frac{1}{3} \quad q = \frac{2}{3}$

$\frac{1}{3} = \frac{2}{6}$

${}_{10}C_6 \left(\frac{1}{3}\right)^6 \left(\frac{2}{3}\right)^{10-6 = 4}$

$.0569$

$= 5.69\%$

3. Kate took a multiple choice test with four choices for each question. If she guessed last 5 questions, what is the probability that she got **exactly** three questions correct?

$n = 5, \quad r = 3, \quad p = \frac{1}{4}, \quad q = \frac{3}{4}$

${}_5C_3 \left(\frac{1}{4}\right)^3 \left(\frac{3}{4}\right)^{5-3=2}$

$= .0879$

8.79%

4. Chad randomly chooses a card from a standard deck 30 times. What is the proability that he gets a diamond **exactly** four times?

5. There are 30 balls numbered 1-30 in a bag. One is randomly selected 12 times. What is the probability of getting a prime number **exactly** five times?	**6.** The probability of customers at a store receiving a coupon printed on their receipt is 1 in 8. If 20 customers visit the store today, what is the probability that **exactly** three of them receive coupons?
7. In a game of chance, the probability of winning is 30%. If 15 games are played, find the probability of winning **exactly** six games.	**8.** A certain traffic light is red for 20 seconds, yellow for 4 seconds, and green for 36 seconds. Find the probability that of the next 12 cars that randomly arrive at the light, **exactly** 4 will be stopped by a red light.

Examples

(with at least, at most, etc.)

9. Kyle tosses a coin 10 times. What is the probability that he gets tails **at least** 8 times?

$n=10$ $r=8, 9, 10$ $p=\frac{1}{2}$ $q=\frac{1}{2}$

$\frac{8}{10}$ times $\rightarrow {}_{10}C_8\left(\frac{1}{2}\right)^8\left(\frac{1}{2}\right)^2 = .0439$

$\frac{9}{10} \rightarrow {}_{10}C_9\left(\frac{1}{2}\right)^9\left(\frac{1}{2}\right)^1 = .00977$

$\frac{10}{10} \rightarrow {}_{10}C_{10}\left(\frac{1}{2}\right)^{10}\left(\frac{1}{2}\right)^0 = .00976$

$+$

$.05468$

5.47%

10. Carol randomly chooses a day of the week six times. What is the probability of choosing Tuesday or Thursday **at most** three times?

$n=6, r=3, 2, 1, 0$ $p=\frac{2}{7}$ $q=\frac{5}{7}$

$\frac{3}{6} \rightarrow {}_6C_3\left(\frac{2}{7}\right)^3\left(\frac{5}{7}\right)^3 = .16999$

$\frac{2}{6} \rightarrow {}_6C_2\left(\frac{2}{7}\right)^2\left(\frac{5}{7}\right)^4 = .3187$

$\frac{1}{6} \rightarrow {}_6C_1\left(\frac{2}{7}\right)^1\left(\frac{5}{7}\right)^5 = .3187$

$\frac{0}{6} \rightarrow {}_6C_0\left(\frac{2}{7}\right)^0\left(\frac{5}{7}\right)^6 = .13281$

94.02%

11. The soda company prints prizes inside their bottle caps. The probability of winning a prize is 1 in 3. Out of eight bottle caps, what is the probability that **at least** five are winners?

$n=8$ $r=5$ $p=\frac{1}{3}$ $q=\frac{2}{3}$

$\rightarrow {}_8C_5$

$\rightarrow {}_8C_6$

$\rightarrow {}_8C_7$

$\rightarrow {}_8C_8$

$+$

$.088$

8.8%

12. There are 7 boys and 5 girls on the debate team. Each week, one is chosen at random to the lead the debate. For the next 10 weeks, what is the probability that **no more than** four girls are chosen to lead?

$n=10$ $r=4$ $p=\frac{5}{12}$ $q=\frac{7}{12}$

$\rightarrow {}_{10}C_0$

$\rightarrow {}_{10}C_1$

$\rightarrow {}_{10}C_2$

$\rightarrow {}_{10}C_3$

$\rightarrow {}_{10}C_4$

$+$

$.1222$

12.22%

Name: _____ **Unit 11:** Probability & Statistics _____

Date: _____ **Bell:** _____ **Homework 4:** Binomial Theorem & Binomial Probability

**** This is a 2-page document! ****

Directions: Expand each binomial using the binomial theorem.

1. $(a+b)^9$

2. $(m-2)^8$

3. $(3k-1)^6$

$1(3k)^6(-1)^0 \quad 6(3k)^5(-1)^1 \quad 15(3k)^4(-1)^2 \quad 20(3k)^3(-1)^3 \quad 15(3k)^2(-1)^4 \quad 6(3k)^1(-1)^5 \quad 1(3k)^0(-1)^6$

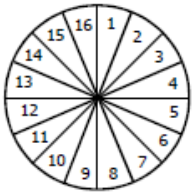
$_6C_0 \quad _6C_1 \quad _6C_2 \quad _6C_3 \quad _6C_4 \quad _6C_5 \quad _6C_6$

$729k^6 - 1458k^5 + 1215k^4 - 540k^3 + 135k^2 - 540k^1 + 135$

4. $(2r+5s)^5$

Directions: Find each binomial probability.	
5. The probability that Ashley wins a race against Madelyn is 25%. What is the probability that Ashley wins **exactly** three of the next five races against Madelyn? $n = 5$, $r = 3$, $p = .25$ $q = .75$ ${}_5C_3 (.25)^3 (.75)^2$ $(5-3)$ use calc: $= .088 \rightarrow 8.8\%$	**6.** A coin is tossed 14 times. What is the probability of getting heads **exactly** 5 times?

7. The spinner below is divided into 16 equally spaced sections. If the spinner is spun 20 times, what is the probability of getting a perfect square **exactly** seven times?



8. The probability of hitting a target is 1/8. What is the probability of hitting the target **exactly** twice in five tries?

9. A die is rolled eight times. What is the probability of getting **exactly** three 6's?

10. There are five blue crayons, seven yellow crayons, and eight red crayons in a box. If one is randomly drawn and replaced 15 times, find the probability of drawing **exactly** four blue crayons.

11. In hockey, Ed makes 7 goals for every 10 shots. If he takes 6 shots, what is the probability that he will make **at least** 5 goals? $n=6, r=5,6$ $p=\frac{7}{10}, q=\frac{3}{10}$

+

12. The probability that it will snow on the last day of January is 85%. If the probability remains the same for the first eight days of February, what is the probability that it will snow **at least** five of those days in February?

$n=8, r=5,6,7,8$ $p=.85$ $q=.15$

+

13. Whenever Nicole rents a movie from iTunes, the probability that it will be a comedy is 52%. Of the next seven movies she rents, what is the probability that she rents **no more than** two comedies?

14. The chance of winning a certain game at a carnival is 2 in 5. If Andy plays the game 12 times, what is the probability that he loses **at most** 3 times?