Conditional Probability

- Given events A and B, conditional probability is the probability of event B occuring, knowing that event A has already occurred.
- Notation: $P(B|A)$
- Read as: the probability of B given A

Conditional Probability Formula: $P(B|A) = \frac{P(A \text{ and } B)}{P(A)}$

Examples

1. Mitchell drew a card from a standard deck of playing cards. What is the probabilitiy that he drew a queen, given that the card was red?

$\frac{2}{26} = \frac{1}{13}$ ✓

2. A number from 1-100 is randomly selected. What is the probability that it is a perfect square, given that it is an odd number?

$\frac{1}{10} = \frac{5}{50}$ (5 square odds)

3. There are 62 people that take yoga class and 48 people that take spinning class at the gym. Fifteen people that take yoga also take spinning. If a person from these groups is selected at random, find the probability that he or she is takes yoga class, if you know they take spinning class.

$\frac{15}{48}$ spin & yoga / spin

yoga	spin & yoga	spin
47	15	33

$\frac{5}{16}$

4. Of the 30 students in Mrs. Smith's math class, 13 are athletes and 9 are in band. Three of the athletes are also in band. If one student is chosen at random, find the probability that this student is in band, if it is known that they are not an athlete.

athlete	athlete & band	band
10	3	6

neither = 11

$\frac{6}{17}$ not athlete

Two-Way Tables

A table that records data that pertains to two different categories.

Example: The table shows the results of a poll of 50 randomly selected students who were asked whether they prefer to watch hockey or football. Find each probability.

	Boys	Girls
Prefer Football	18	6
Prefer Hockey	10	16

a) $P(\text{prefer football} | \text{girl})$

$\frac{6}{22} = \frac{3}{11}$

b) $P(\text{boy} | \text{hockey})$

$\frac{10}{26} = \frac{5}{13}$

Directions: Find each probability using the table.

5. The table shows the results of a poll of 150 randomly selected middle school students who were asked if they take French or Spanish.

	French	Spanish	
6th Grade	19	23	42
7th Grade	25	12	37
8th Grade	38	33	71
	82	68	150

a) P(a 7th grader who takes French)

$\frac{25}{150} = \frac{1}{6}$

b) P(8th grader | takes Spanish)

$\frac{33}{68}$

c) P(takes French | 6th grader)

$\frac{19}{42}$

d) P(7th grader | takes French)

$\frac{25}{82}$

6. The table shows the results of a poll of randomly selected juniors and seniors who were asked if they attended prom.

		Juniors	Seniors	
Prom	**Yes**	28	97	125
	No	56	19	75
		84	116	200

a) P(a junior that did not attend prom)

$\frac{56}{200} = \frac{7}{25}$

b) P(did not attend prom | senior)

$\frac{19}{116}$

c) P(junior | attended prom)

$\frac{28}{125}$

Frequencies

- **Joint Relative Frequencies:** The values in each category divided by total number of values.
- **Marginal Relative Frequencies:** Found by totaling each row and column.

Examples

7. The table below shows the number of first class and coach class passengers on a plane who checked one bag, two bags, or no bags. Find the joint relative and marginal relative frequencies, then find each probability.

	First Class	Coach Class
2 bags	16/160	52/160
1 bag	12/160	60/160
0 bags	2/160	18/160

total = 160

	First Class	Coach Class	Total
2 bags	10%	32.5%	42.5%
1 bag	7.5%	37.5%	45%
0 bags	1.25%	11.25%	12.5%
Total	18.75%	81.25	100%

a) P(a coach class passenger that did not check any bags)

11.25%

b) P(a first class passenger that checked at least one bag)

1 bag + 2 bags

7.5% + 10 = 17.5%

c) P(coach class | checked one bag)

37.5 / 45 = .83 = 83.3%

d) P(checked two bags | first class)

53 / 18.75 = 10 / 18.75 = 53.3%

Date: _____ **Bell:** _____ **Homework 3:** Conditional Probability

** This is a 2-page document! ** Do ALL

Use for questions 1-2: A bucket contains 50 lottery balls numbered 1-50. One is drawn at random. Find each probability.

1. P(multiple of 6 | 2-digit number)

6, 12, 18, 24, 30, 36, 42, 48 . 10 - 50

7/41

2. P(at least 20 | prime number)

3. Marti rolls two dice. What is the probability that the sum of the dice is 7, given that the first die is showing a 2?

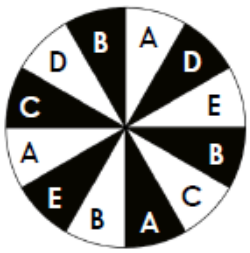
4. Blake randomly chose a letter from alphabet. What is the probability that this letter has at least one line of symmetry, given that it is a consonant?

5. A card is randomly selected from a standard deck of playing cards. Find the probability that it is a face card, given that a black card is drawn.

6. **A month of the year is randomly chosen. Find the probability that it has no more than 30 days, given that it starts with the letter A.**

$\frac{1}{2}$

Use for 7-9: The wheel below is spun. Find each probability.



7. P(black | A)

$\frac{1}{3}$

8. P(C | white)

$\frac{1}{6}$

9. P(black | B or E)

$\frac{3}{5}$

10. Out of the 125 children at summer camp, 45 signed up for swimming and 38 signed up for arts and crafts. Twelve students who signed up for swimming also signed up for arts and crafts. If a child is randomly selected, what is the probability that they are signed up for swimming, if it is known that they did not sign up for arts and crafts?

33 swim + not art

87 not in art

45 swim 38 Art

33 12 26

125

71

54 neither

11. Out of the 56 players on the football team, 24 are on honor roll and 18 have perfect attendance. Seven who are on honor roll also have perfect attendance. If a player is chosen at random, what is the probability that they are on honor roll, if it is known that they also have perfect attendance?

12. The table below shows the number of students that do or do not have their own car and whether they have part time jobs.

		Part Time Job	
		Yes	No
Car	Yes	78	18
	No	30	24

a) P(a student with a part-time job without a car) $\frac{30}{150} = \frac{1}{5}$

b) P(no car| does not have a part-time job) $\frac{24}{42} = \frac{4}{7}$

c) P(part-time job| car) $\frac{78}{96} = \frac{13}{16}$

13. The table below shows the fate of the first, second, and third class passengers on the Titanic.

	Survived	Died
1st Class	199	120
2nd Class	117	155
3rd Class	172	537

a) P(a 3rd class survivor)

b) P(1st class | died)

c) P(survived | 2nd class)

d) P(1st or 3rd class | survived)

14. The table below represents the GPA of a group of undergrad students and whether or not they will be attending grad school. Find the joint relative and marginal relative frequencies.

		Attending Grad School	
		Yes	No
GPA	≥ 3.0	96	18
	< 3.0	32	54

		Attending Grad School		
		Yes	No	Total
GPA	≥ 3.0			
	< 3.0			
Total				

If a student is chosen at random, find each probability:

a) P(not attending grad school)

b) P(< 3.0 GPA and not attending grad school)

c) P(not attending grad school | ≥ 3.0 GPA)

d) P(< 3.0 GPA | attending grad school)