Name:____________________ Period:____

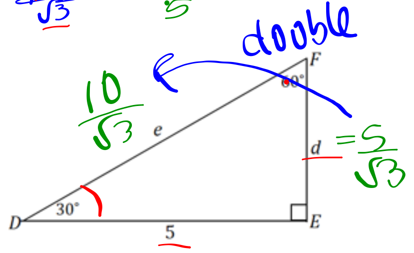
CH5 Review: Using Trigonometry to Find Side Lengths of a Triangle

Classwork

a. Find the lengths of d and e.

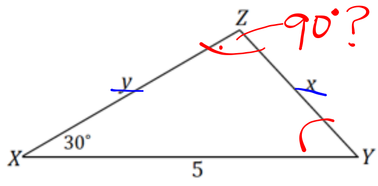
w/calc $5 \cdot \tan 30° = \frac{d}{5}$
$2.886 = d = \frac{5}{\sqrt{3}}$

w/out calc $5 \cdot \tan 30°$
$5 \cdot \frac{1}{\sqrt{3}} = d = \frac{5}{\sqrt{3}}$ ✓

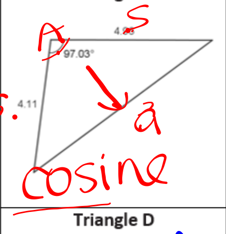
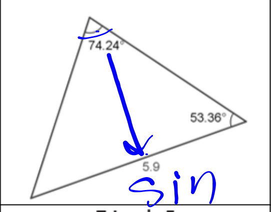
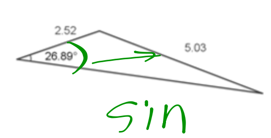
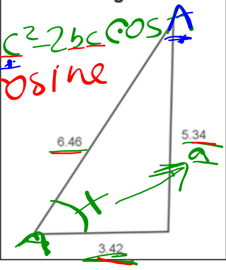
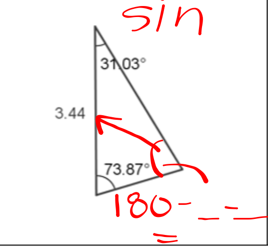
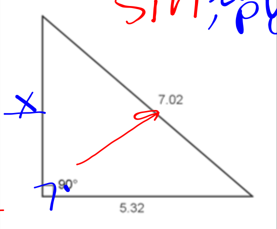


b. Find the lengths of x and y. How is this different from part (a)?

because it's not a right △, we can't use the trig ratios. Thus law of sines & cosines was invented!



c. For each triangle shown below, decide whether you should use the law of sines, the law of cosines, or neither to begin finding the missing measurements. Explain how you know.

Triangle A	Triangle B	Triangle C
 cosine	 sin	 sin
Triangle D	**Triangle E**	**Triangle F**
 $a^2=b^2+c^2-2bc\cos A$ cosine	 sin; 180 − _ − _	 sin, or pythagorean theorem

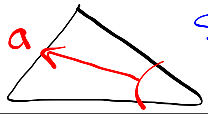
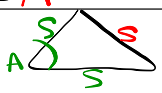
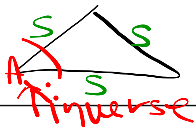
d. What types of given information will help you to decide which formula to use to determine missing measurements? Summarize your ideas in the table shown below:

sin: we need a matching pair (an angle & it's opposite side) $\frac{\text{side}}{\text{angle}} = \frac{x}{\text{angle}}$

cos: No matching pair
if your 3 out 4 of the variables then you can solve for the 4th.
Side-angle-side

Determining Missing Measurements

Notes Blue Packet.

Given Measurements	Formulas to Use
Right Triangle 2 sides → angle angle + side → other side 2 sides → other side	Trigonometry Functions $\sin\theta = \frac{O}{H}$ $\sin^{-1}\left(\frac{O}{H}\right) = \theta$ $\cos\theta = \frac{A}{H}$ inside angle → inverse $\tan\theta = \frac{O}{A}$ Pythagorean Theorem $a^2 + b^2 = c^2$
Non-Right Triangle Opposite angle and side pair  	Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ (sides) (angles) * flip if finding Angles
Non-Right Triangle S-A-S → side  S-S-S → angle  	Law of Cosines $a^2 = b^2 + c^2 - 2bc\,\text{Cos}(A)$

Given Measurements	Formulas to Use
Right Triangle *Two side measurements* *One angle and one side measurement*	Trigonometry Functions $\sin(\theta) = \frac{O}{H}$ $\cos(\theta) = \frac{A}{H}$ $\tan(\theta) = \frac{O}{A}$ *Where O is the leg opposite θ, A is the leg adjacent to θ, and H is the hypotenuse* Pythagorean Theorem $a^2 + b^2 = c^2$ *Where a and b are legs of a right triangle and c is the hypotenuse*
Non-Right Triangle *Any two angles and one side* *Two sides and the angle opposite one of them*	Law of Sines $\frac{\sin(A)}{a} = \frac{\sin(B)}{b} = \frac{\sin(C)}{c}$ *Where a is opposite angle A, b is opposite angle B, and c is opposite angle C*
Non-Right Triangle *Three sides because this formula relates all three sides of a triangle* *Two sides and the angle between them because the law of sines requires an angle and the opposite side*	Law of Cosines $a^2 = b^2 + c^2 - 2bc\cos(A)$ *Where A is the measure of the angle opposite side a*

Example 1

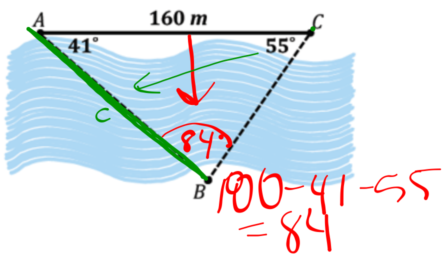
A surveyor needs to determine the distance between two points A and B that lie on opposite banks of a river. A point C is chosen 160 meters from point A, on the same side of the river as A. The measures of $\angle BAC$ and $\angle ACB$ are 41° and 55°, respectively. Approximate the distance from A to B to the nearest meter.

$\frac{\sin 55 \times 160}{\sin 84} = \frac{c}{\sin 55}$

$\frac{131.064}{\sin 84} = c$

$131.78 = c$

$132 = c$

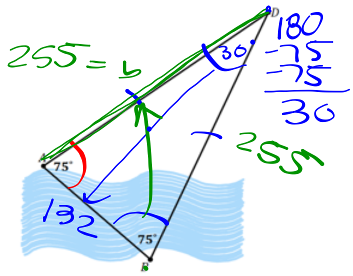


Example 2

Our friend the surveyor from Example 1 is doing some further work. He has already found the distance between points A and B (from Example 1). Now he wants to locate a point D that is equidistant from both A and B and on the same side of the river as A. He has his assistant mark the point D so that $\angle ABD$ and $\angle BAD$ both measure 75°. What is the distance between D and A to the nearest meter?

$\sin 75 \cdot \frac{132}{\sin 30} = \frac{b}{\sin 75}$

$255 = b$

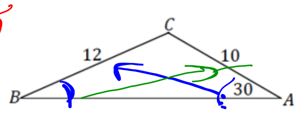


1. In $\triangle ABC$, $m\angle A = 30$, $a = 12$, and $b = 10$. Find $\sin\angle B$. Include a diagram in your answer.

$\frac{10 \cdot \sin 30}{12} = \frac{\sin B}{10}$

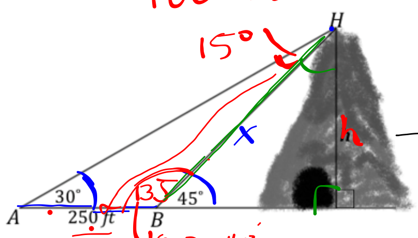
$\sin^{-1} .41 = \sin B$

$24.6 = B$



2. A car is moving toward a tunnel carved out of the base of a hill. As the accompanying diagram shows, the top of the hill, H, is sighted from two locations, A and B. The distance between A and B is 250 ft. What is the height, h, of the hill to the nearest foot?

$180 - 135 - 30 = 15$



$\sin 30 \cdot \frac{250}{\sin 15} = \frac{x}{\sin 30}$

$482.96 = x$

S.O.H.

$482.96 \cdot \sin 45 = \frac{h}{482.96}$

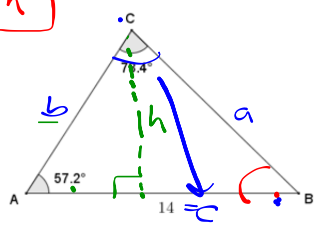
$342 = h$

3. Given $\triangle ABC$, $AB = 14$, $\angle A = 57.2°$, and $\angle C = 78.4°$, calculate the measure of angle B to the nearest tenth of a degree, and use the law of sines to find the lengths of $\overline{AC}$ and $\overline{BC}$ to the nearest tenth.

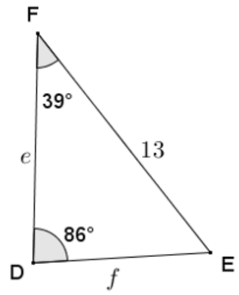
Calculate the AREA of $\triangle ABC$ to the nearest square unit.

b. Area $= \frac{1}{2}$ base $\cdot$ height

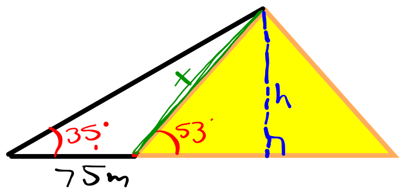
$\frac{1}{2}(14) \cdot$ ____



4. Given $\triangle DEF$, $\angle F = 39°$, and $EF = 13$, calculate the measure of $\angle E$, and use the law of sines to find the lengths of $\overline{DF}$ and $\overline{DE}$ to the nearest hundredth.

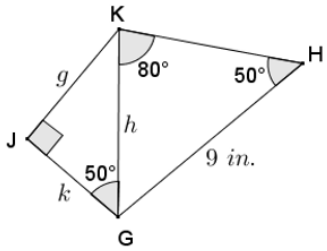


5. At the base of a pyramid, a surveyor determines that the angle of elevation to the top is 53°. At a point 75 meters from the base, the angle of elevation to the top is 35°. What is the distance from the base of the pyramid up the slanted face to the top? X



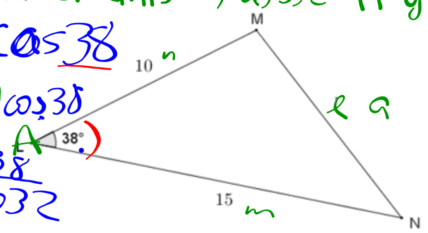
★Bonus★ find the height of the pyramid.

6. Given quadrilateral $GHKJ$, $m\angle H = 50°$, $m\angle HKG = 80°$, $m\angle KGJ = 50°$, $m\angle J$ is a right angle, and $GH = 9$ in., use the law of sines to find the length of $\overline{GK}$, and then find the lengths of $\overline{GJ}$ and $\overline{JK}$ to the nearest tenth of an inch.



7. Given triangle LMN, $LM = 10$, $LN = 15$, and $m\angle L = 38°$, use the law of cosines to find the length of $\overline{MN}$ to the nearest tenth.

you can re-label this w/ a,b,c if you want.
l,m,n
a,b,c.



$l^2 = m^2 + n^2 - 2mn\cos 38$

$l^2 = 15^2 + 10^2 - 2(15)(10)\cos 38$

$225 + 100 - 300\cos 38$

$325 - 236.4032$

$\sqrt{l^2} = \sqrt{88.6}$

$l = 9.41$