

Notes

$$f(x) = \frac{ax^n + \dots}{bx^m + \dots}$$

← nth degree polynomial
← mth degree polynomial

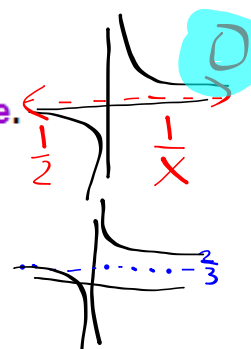
1 If $n < m$, *proper*, then the x-axis is the horizontal asymptote.

2 If $n = m$, then the horizontal asymptote is the line

$$y = \frac{a}{b}$$

$$\frac{2x}{3(x+1)} = y = \frac{2}{3}$$

3 If $n > m$, then there is no horizontal asymptote. (There is a slant diagonal or oblique asymptote.)



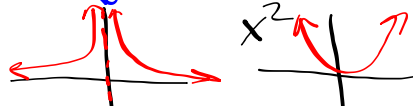
odd/even asymptotes.

A. $\frac{1}{x^1} = 0$

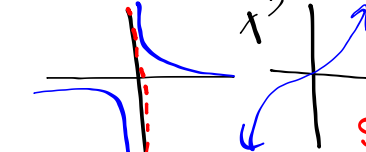


odd exponents
opposite directions
on each side of the asymptote

B. $\frac{1}{x^2} = 0$

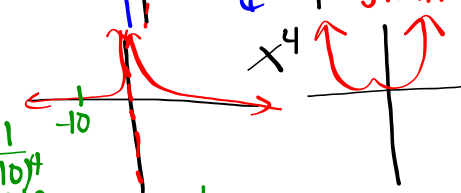


C. $\frac{1}{x^3} = 0$



even exponents (in the denominator)
same direction
- just like "even Stevens"
similar to a U shape, up on both sides.

d. $\frac{1}{x^4} = 0$



$\oplus = \frac{1}{(-10)^4}$
graphs are always \oplus unless there is a negative out front!

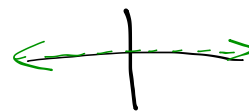


HA: end behavior

same
same $\frac{2x^2}{3x^2} = \frac{2}{3} = y$



small
big $\frac{x}{x^2} = y = 0$



big
small $\frac{x^2}{x^1}$

No Horizontal, but.....

DIVIDE.
Approaches,
what is left
after division!

$x \sqrt{\frac{x}{x^2}} = y = x$

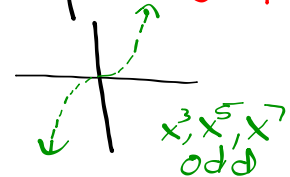


$x \sqrt{\frac{x^2}{x^3}} = y = x^2$



$\frac{x^7}{x^4} = x^3$

$x^4 \sqrt{\frac{x^3}{x^7}} = y = x^3$



Quick notes

$\frac{0}{\#} = 0$

$\frac{\#}{0} = \text{undefined}$

Chapter 3 and 4 Review

Period__

Multiply.

1) $(2a-7)(a+5)$

$$2a^2 + 10a - 7a - 35$$

$$\boxed{2a^2 + 3a - 35}$$

3) $(4x+4y)(8x-4y)$

2) $(2v-7)(5v-5)$

4) $(3m-8n)(2m+3n)$

Remainder Theorem: Evaluate each function at the given value. Yes/ No, is it a root?

5) $f(a) = a^3 - 5a^2 + 3a + 3$ at $a = 2$

$$(2)^3 - 5(2)^2 + 3(2) + 3 = 8 - 20 + 6 + 3 = -3 \neq 0$$

~~NO!~~

6) $f(a) = a^3 - 3a^2 - 10a - 5$ at $a = -2$

Find all zeros. State whether it would bounce or pass at each zero(multiplicity).

7) $f(x) = (x-5)(x-2)(x+2)$

$$5 \quad 2 \quad -2$$

8) $f(x) = (x+3)(x-1)(x+1)$

9) $f(x) = x(x+1)(x^2+1) = 0$

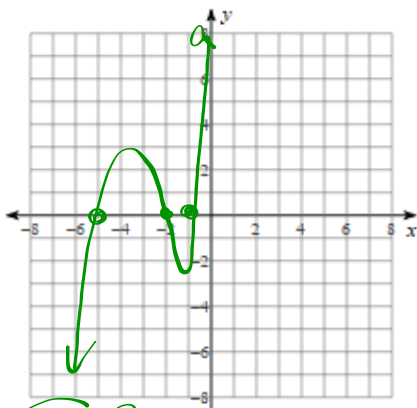
$$0 \quad -1 \quad \sqrt{x^2+1}$$

$$\boxed{x = \pm i}$$

10) $f(x) = (x-1)(x+1)(x-2)(x+2)(x^2+4)$

Use long division to factor completely, then graph.

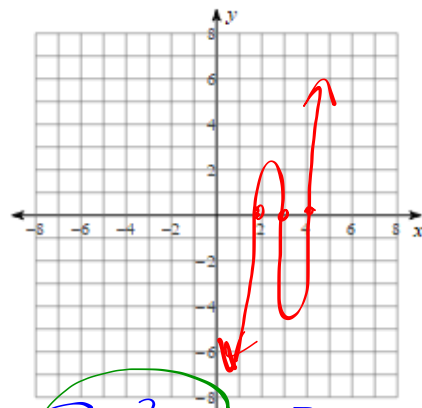
11)



$$\begin{array}{r}
 x^2 + 3x + 2 \\
 x + 5 \overline{) x^3 + 8x^2 + 17x + 10} \\
 \underline{-x^3 - 5x^2} \\
 3x^2 + 17x + 10 \\
 \underline{-3x^2 - 15x} \\
 2x + 10 \\
 \underline{-2x - 10} \\
 0
 \end{array}$$

$(x+5)(x^2+3x+2)$
 $(x+5)(x+2)(x+1)$
 $\begin{matrix} 2 & & 1 \\ \diagdown & & / \\ & 3 & \\ \diagup & & \diagdown \\ 2 & & 1 \end{matrix}$
 $\begin{matrix} -2 & -1 & -5 \end{matrix}$

12)



$$\begin{array}{r}
 x^2 - 6x + 8 \\
 x - 3 \overline{) x^3 - 9x^2 + 26x - 24} \\
 \underline{-x^3 + 3x^2} \\
 -6x^2 + 26x - 24 \\
 \underline{+6x^2 - 18x} \\
 8x - 24 \\
 \underline{-8x + 24} \\
 0
 \end{array}$$

$(x-3)(x^2-6x+8)$
 $(x-3)(x-4)(x-2)$
 $\begin{matrix} -4 & & -2 \\ \diagdown & & / \\ & -6 & \\ \diagup & & \diagdown \\ -4 & & -2 \end{matrix}$
 $\begin{matrix} 4 & 2 & 3 \end{matrix}$

Simplify each expression.

13) $\frac{4}{(3x+18)} + \frac{4(x+6)}{3(x+6)}$
 $\frac{4 + 4x + 24}{3(x+6)} = \frac{4x + 28}{3(x+6)}$
 $\frac{4(x+7)}{3(x+6)}$

14) $\frac{4}{(m^2-3m)} + \frac{5m}{(3m^2-9m)}$
 $\frac{4m^2 - 12m + 5m}{3(m^2-3m)} = \frac{4m^2 - 7m}{3(m^2-3m)}$

15) $\frac{7}{n-6} + \frac{6}{n-3}$

$\frac{160}{10 \cdot 16} = \frac{160}{-7}$

$\frac{(n+8)(n-5)}{(n+2)(n+2)} - \frac{5n(n+2)}{(n+8)(n+2)}$

$\frac{n^2 + 3n - 40 - (5n^2 + 10n)}{(n+8)(n+2)} = \frac{-4n^2 - 7n - 40}{(n+8)(n+2)}$

17) $\frac{(a^2 - 5a - 50)}{a^2 + 6a + 5}$

18) $\frac{p^2 + 7p + 12}{3p^3 + 9p^2}$

19)
$$\frac{5n^2}{35n^3 - 20n^2} \cdot \frac{49n - 28}{10}$$

20)
$$\frac{1}{8x} \div \frac{x+8}{8x^2 + 16x}$$

21)
$$\frac{x^2 + 4x - 21}{x^2 + 7x - 30} \times \frac{x^2 + 12x + 20}{x^2 - x - 6}$$

22)
$$\frac{3x^2 - 7x + 4}{4x^2 - x - 3}$$

Match the following functions with the corresponding graph. *bounce!*

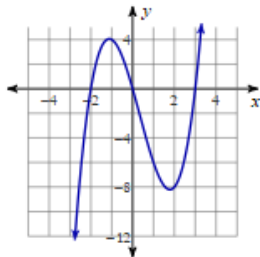
23) i. $x(x+2)(x-3)$

ii. B $(x-2)^2(x+3)$

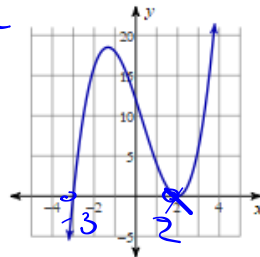
iii. $(x-2)(x+3)^2$

iv. $x^2(x-3)$

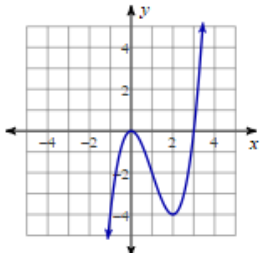
A)



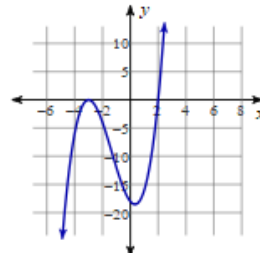
B)



C)

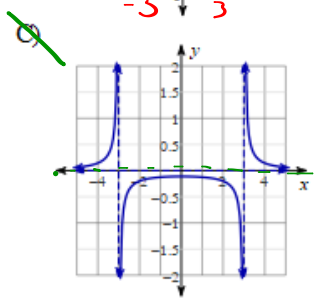
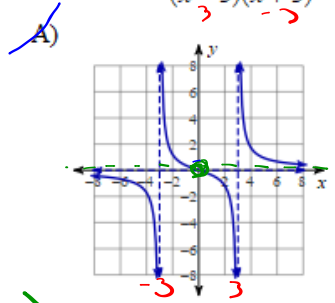


D)



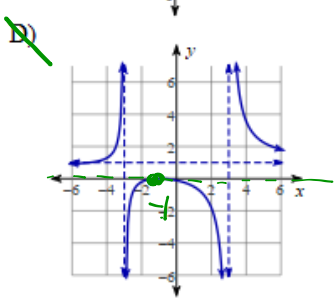
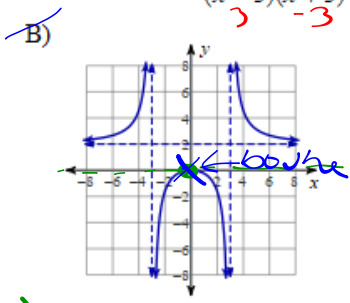
24) i. C $\frac{1}{(x-3)(x+3)} = 0$ none
 3 -3 ↘ bounce

iii. B $\frac{2x^2}{(x-3)(x+3)} = 0$



ii. A $\frac{3x}{(x-3)(x+3)} = 0$

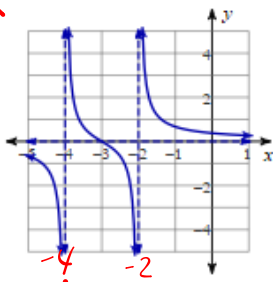
iv. D $\frac{(x+1)^2 - 1}{(x-3)(x+3)}$



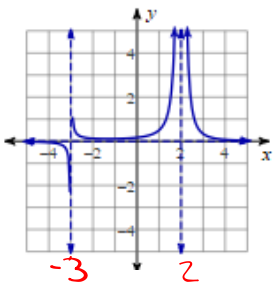
25) i. $\frac{2}{(x-2)^2(x+3)}$

iii. $\frac{3}{x^2-3x-10}$

~~A)~~



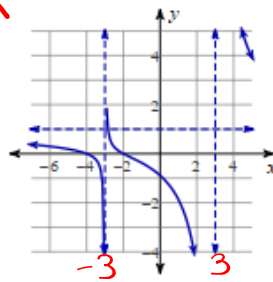
C)



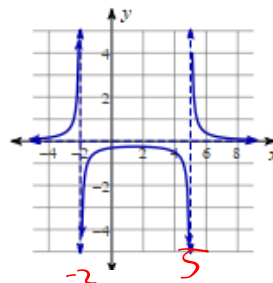
ii. ~~A~~ $\frac{x+3}{x^2+6x+8}$
 $(x+2)(x+4)$
 $\begin{array}{r} 4 \\ \times 2 \\ \hline 8 \end{array}$

iv. ~~B~~ $\frac{x^2+6x+8}{(x-3)(x+3)} = 0$

~~B)~~



D)



26) Determine the features and graph:

$$\frac{x+1}{(x-3)(x+3)} = 0$$

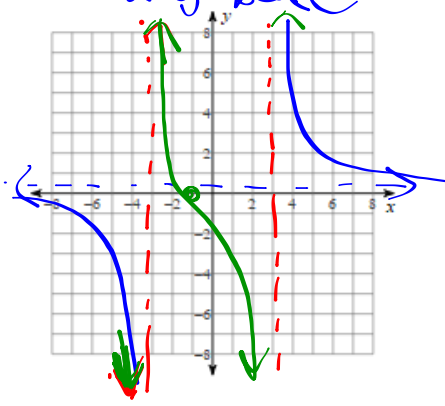
3 -3

x-intercept(s): -1

Vertical Asymp.: 3, -3

End Behavior Asymp.: 0 $\frac{x}{x^2}$ ↓

Horizontal asymptote



27) Determine the features and graph:

$$\frac{x+2}{x^2+6x+5}$$

x-intercept(s): _____

Vertical Asymp.: _____

End Behavior Asymp.: 0

