8.3 Y B Normal?

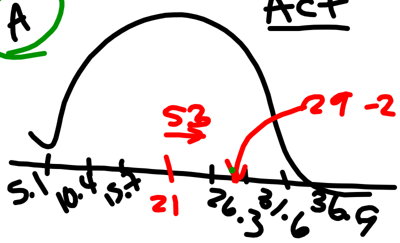
A Solidify Understanding Task

As a college admissions officer, you get to evaluate hundreds of applications from students that want to attend your school. Many of them have good grades, have participated in school activities, have done service within their communities, and all kinds of other attributes that would make them great candidates for attending the college you represent. One part of the application that is considered carefully is the applicants score on the college entrance examination. At the college you work for, some students have taken the ACT and some students have taken the SAT.

You have to make a final decision on two applicants. They are both wonderful students with the very same G.P.A. and class rankings. It all comes down to their test scores. Student A took the ACT and received a score of 29 in mathematics. Student B took the SAT and received a score of 680 in mathematics. Since you are an expert in college entrance exams, you know that both tests are designed to be normally distributed. A perfect ACT is 36. The ACT mathematics section has a mean of 21 and standard deviation of 5.3. (Source: National Center for Education Statistics 2010) A perfect score on the SAT math section is 800. The SAT mathematics section has a mean of 516 and a standard deviation of 116. (Source: www.collegeboard.com 2010 Profile).

1. Based only on their test scores, which student would you choose and why?

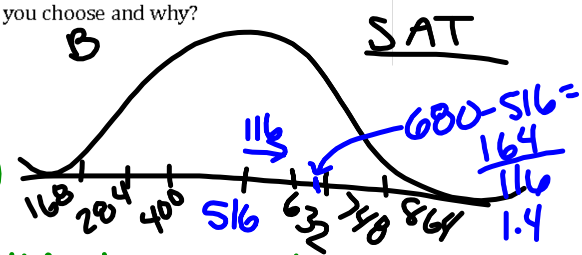
A Act



z-score

$29 - 21 = \frac{8}{5.3} = 1.509$

B SAT



$680 - 516 = \frac{164}{116}$ 1.4

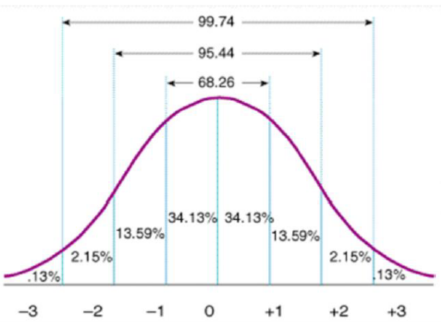
Act, student A, scored more above the average than student B.

This analysis is starting to make you hungry, so you call your friend in the Statistics Department at the university and ask her to go to lunch with you. During lunch, you tell her of your dilemma. The conversation goes something like this:

You: I'm not sure that I'm making the right decision about which of two students to admit to the university. Their entrance exam scores seem like they're in about the same part of the distribution, but I don't know which one is better. It's like trying to figure out which bag of fruit weighs more when one is measured in kilograms and one is measured in pounds. They might look like about the same amount, but you can't tell the exact difference unless you put them on the same scale or convert them to the same units.

Statistician: Actually, there is a way to make comparisons on two different normal distributions that is like converting the scores to the same unit. The scale is called the "standard normal distribution". Since it was invented to make it easy to think about a normal distribution, they set it up so that the mean is 0 and the standard deviation is 1.

Here's what your statistician friend drew on her napkin to show you the standard normal distribution:



You: Well, that looks just like the way I always think of normal distributions.

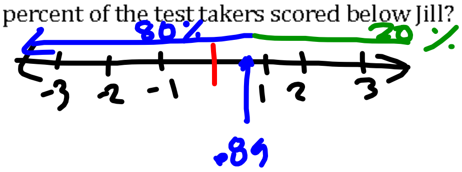
Statistician: Yes, it's pretty simple. When we use this scale, we give things a z-score. A z-score of 1 means that it's 1 standard deviation above the mean. A z-score of -1.3 means that it is between 1 and 2 standard deviations below the mean. Easy-peasy.

What's even better is that when we have a z-score there are tables that will show the area under the curve to the left of that score. For a test score like the ACT or SAT, it shows the percentage of the population (or sample) that is below that score. I've got a z-score table right here in my purse. See, the z-score is -1.3, then 9.68% of the population scored less. You can also say that 90.32% of the population scored better, so -1.3 wouldn't be a very good score on a test.

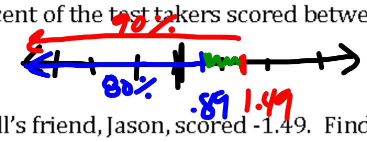
Try it: Let's say you had two imaginary test takers, Jack and Jill. Jack's z-score was 1.49 and Jill's z-score was 0.89.

2. What percent of the test takers scored below Jack? What percent scored above Jack? 10%

3. What percent of the test takers scored below Jill? What percent scored above Jill? 20%



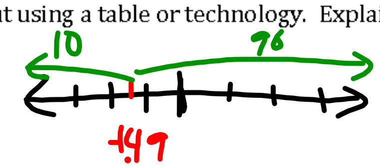
4. What percent of the test takers scored between Jack and Jill?



$$\begin{array}{r} 90 \\ -80 \\ \hline 10\% \end{array}$$

between.

5. Jack and Jill's friend, Jason, scored -1.49. Find the number of test takers that scored above him without using a table or technology. Explain your strategy.



90%

You: That's very cool, but the two scores I'm working with are not given as z-scores. Is there some way that I can transform values from some normal distribution like the scores on the ACT or SAT to z-scores?

Statistician: Sure. The scale wouldn't be so amazing if you couldn't use it for any normal distribution. There's a little formula for transforming a data point from any normal distribution to a standard normal distribution:

z-score = $\frac{data\ point - mean}{standard\ deviation}$

$\frac{\text{score} - \text{average}}{\text{S.D.}} = z$

6. So, if you have an ACT score of 23. The mean score on the ACT is 21 and the standard deviation is 5.2. What would you estimate the z-score to be?

7. Let's use the formula to figure it out: z-score = $\frac{23-21}{5.2}$. How was your estimate? Explain why this value is reasonable.

You: That's great. I'm going back to the office to decide which student is admitted.

8. Compare the scores of Student A and Student B. Explain which student has the highest mathematics test score and why.

Name Statistics | 8.3

Ready, Set, Go!

Ready

Topic: Probability

At South Beach High School, there are 2500 students attending. Mariana surveys 40 of her friends where they prefer to eat lunch. She created the following two-way table showing her results:

	9th Grade	10th Grade	11th Grade	12th Grade	Totals
School Cafeteria	18	6	2	1	**28**
Off Campus	2	4	3	4	**12**
Totals	**20**	**10**	**5**	**5**	**40**

Mariana plans to use her data to answer the following questions:

I. Do students prefer to eat on campus or off campus overall?
II. Is there a difference between grade levels for where students prefer to eat lunch?

1. In Mariana's sample, what percent of students prefer school lunch?

What percent prefer to eat off campus?

2. For each grade level in her sample, determine the percent of students that prefer school lunch and the percent that prefer off campus lunch. Do you notice anything unusual?

3. Based on her sample, Mariana concludes that students at South Beach High school overall like school lunch. Do you agree or disagree? Why?

A company makes a mean monthly income of $20,300 with a standard deviation of $3,200. In one given month the company makes $29,500.

4. Find the z-score.

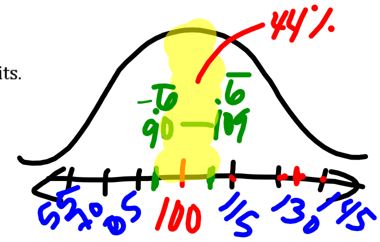
5. Assuming the companies monthly income is Normal, what percent of the time does the company make more than this amount? Less than?

6. What percent of the time does the company make between $15,000 and $25,000?

7. If the company needs to make $16,400 in order to break even, how likely in a given month is the company to make a profit?

Use Desmos link on my website to get percents

On the Wechsler Adult Intelligence Scale, an average IQ is 100 with a standard deviation of 15 units. (Source: http://en.wikipedia.org/wiki/Intelligence_quotient)

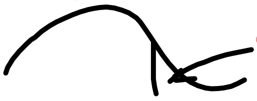


2.3
135

8. IQ scores between 90 and 109 are considered average. Assuming IQ scores follow a Normal distribution, what percent of people are considered average?

guess = 44% desmos 45%

9. One measure of Genius is an IQ score of above 135. What percent of people are considered genius?



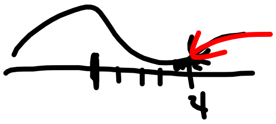
~4.7% guess

desmos ≈ 1%

10. Einstein had an IQ score of 160. What is his z-score?

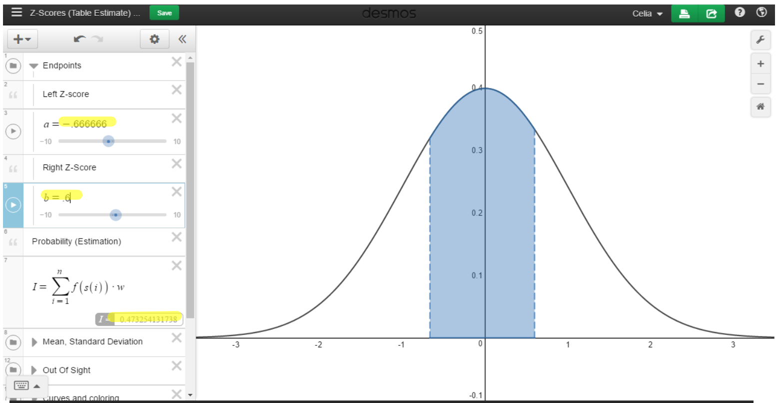
$$\frac{160-100}{15} = \frac{60}{15} = 4$$

11. What is the probability of an individual having a higher IQ than Einstein?

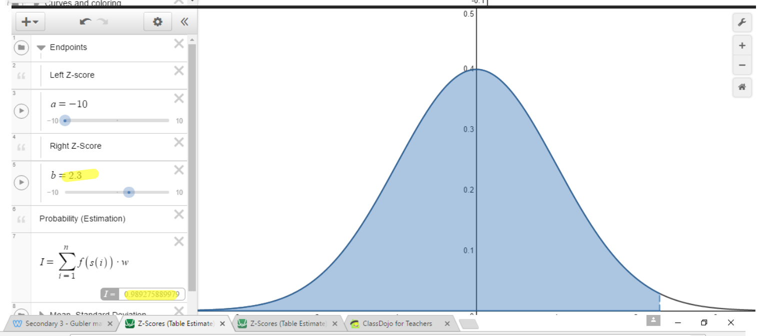


.003% smarter than Einstein.

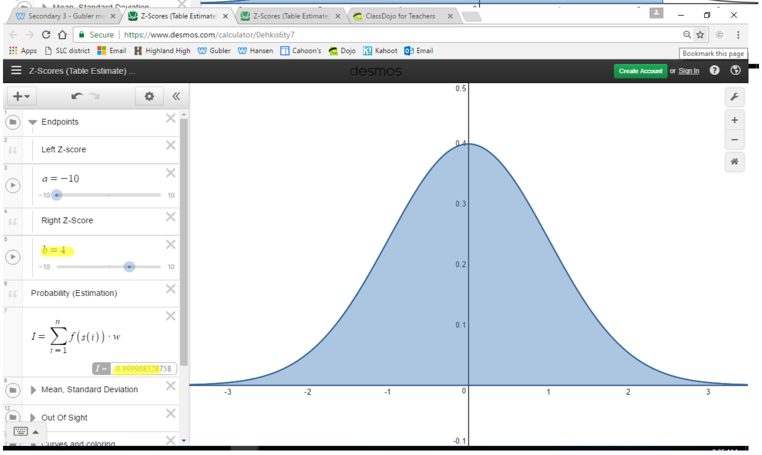
#8


Z-Scores (Table Estimate) ...
Save
Endpoints
Left Z-score
$a = -.666666$
Right Z-Score
$b = .6$
Probability (Estimation)
$I = \sum_{i=1}^{n} f(s(i)) \cdot w$
$I = 0.473254131738$
Mean, Standard Deviation
Out Of Sight

#9


Endpoints
Left Z-score
$a = -10$
Right Z-Score
$b = 2.3$
Probability (Estimation)
$I = \sum_{i=1}^{n} f(s(i)) \cdot w$
$I = 0.989275889979$

#10


Z-Scores (Table Estimate) ...
Endpoints
Left Z-score
$a = -10$
Right Z-Score
$b = 4$
Probability (Estimation)
$I = \sum_{i=1}^{n} f(s(i)) \cdot w$
$I = 0.999968328758$
Mean, Standard Deviation
Out Of Sight

Go

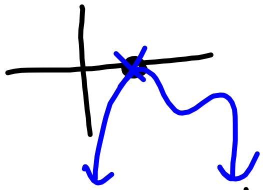
Topic: Sketching Polynomials

Without using technology, sketch the graph of a polynomial function with the given characteristics.

bounce No touch!

12. A quartic function with a leading coefficient of -2 with one double zero and ~~two complex roots~~.

$-2x^4$

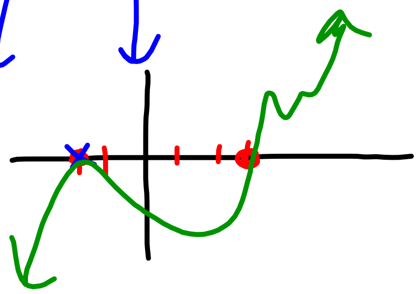


bounce

13. $f(x) = (x+2)^2(x-3)^5$ pass

-2 3

$\sim x^7$



14. $g(x) = -2(x-3)^2(x+5)(2x-5)^3$

touches 3 pts that are positive (random)

15. A cubic function with a leading coefficient of 4 and three positive roots.

$4x^3$