5.6 More Than Right
A Develop Understanding Task

LAST TIME...

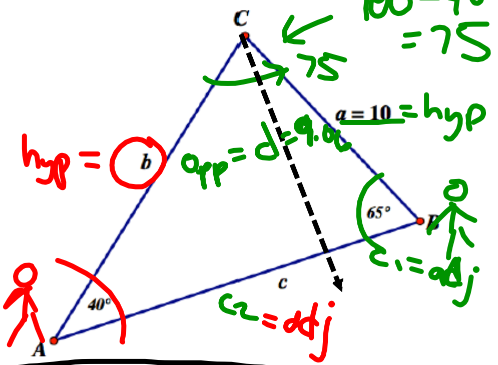
We can use right triangle trigonometry and the Pythagorean theorem to solve for missing sides and angles in a right triangle. What about other triangles? How might we find unknown sides and angles in acute or obtuse triangles if we only know a few pieces of information about them?

In the previous task we found it might be helpful to create right triangles by drawing an altitude in a non-right triangle. We can then apply trigonometry or the Pythagorean theorem to the smaller right triangles, which may help us learn something about the sides and angles in the larger triangle.

SOH CAH TOA

See if you can devise a strategy for finding the missing sides and angles of each of these triangles.

1.



180−40−65 = 75

hyp = b; opp = d = 9.06; a = 10 = hyp; c_1 = adj; c_2 = adj

$10 \cdot \sin 65 = \frac{d}{10}$ d = 9.06

$10 \cdot \cos 65 = \frac{c_1}{10}$ $c_1 = 4.22$

$b \cdot \frac{\sin 40}{\sin 40} = \frac{9.06}{b} \cdot \frac{b}{\sin 40}$

$b = \frac{9.06}{\sin 40}$ $b = 14.09$

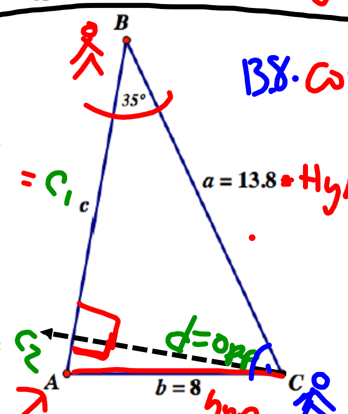
$\tan 40 = \frac{9.06}{c_2}$

$c_2 = \frac{9.06}{\tan 40}$ $c_2 = 10.79$

$c = c_1 + c_2$

$c = 4.22 + 10.79 = 15.01$

2.



adj = c_1; Hyp 13.8; adj = c_2; d = opp; hyp

$13.8 \cdot \cos 35 = \frac{c_1}{13.8}$

$c_1 = 11.3$

$\sin 35 = \frac{d}{13.8}$

d = 7.91

$(c_2)^2 + (7.91)^2 = 8^2$

$(c_2)^2 + 62.6 = 64$

$-62.6 \quad -62.6$

$\sqrt{(c_2)^2} = \sqrt{1.4}$

$c_2 = 1.18$

$C = c_1 + c_2 = 12.48$

* $\cos A = \frac{c_2}{b} = \frac{1.18}{8}$

$A = \cos^{-1}\left(\frac{1.18}{8}\right)$

$A = 81.52°$

* 180 − 81.52 − 35 = 63.48 = ∠C.

5.6 More Than Right

A Develop Understanding Task

REPEAT!!

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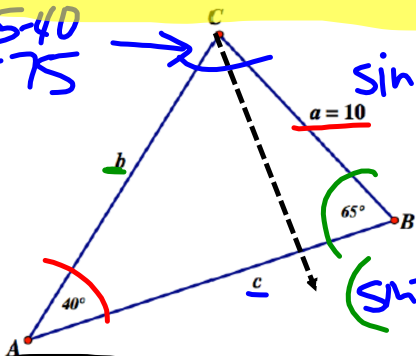
In the previous task we found it might be helpful to create right triangles by drawing an altitude in a non-right triangle. We can then apply trigonometry or the Pythagorean theorem to the smaller right triangles, which may help us learn something about the sides and angles in the larger triangle.

See if you can devise a strategy for finding the missing sides and angles of each of these triangles.

Law of Sines $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$

1.



$180-65-40 = 75$

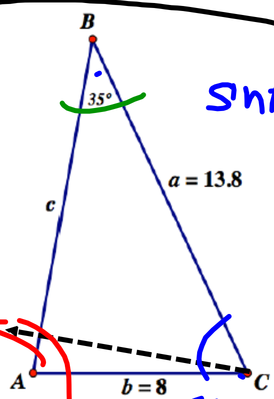
$\sin 65 \cdot \frac{10}{\sin 40} = \frac{b}{\cancel{\sin 65}} \cancel{\sin 65}$

$14.099 = b$

$\left(\sin 75 \cdot \frac{10}{\sin 40}\right) = \frac{c}{\cancel{\sin 75}} \cancel{\sin 75}$

$15.027 = c$

2.



81.66

$\sin A \cdot \frac{8}{\cancel{\sin 35}} = \frac{13.8}{\cancel{\sin A}} \cancel{\sin 35}$

$\sin A \cdot \frac{\cancel{8}}{8} = \frac{13.8 \sin 35}{8}$

$\cancel{\sin} A = \sin^{-1} .989\ldots$

$A = 81.66°$

$180 - 81.66 - 35$

$\angle C = 63.34°$

$\frac{8}{\sin 35} = \frac{c}{\sin 63.34}$

$12.46 = c$

5.7 Justifying the Laws

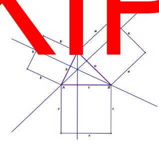
A Solidify Understanding Task

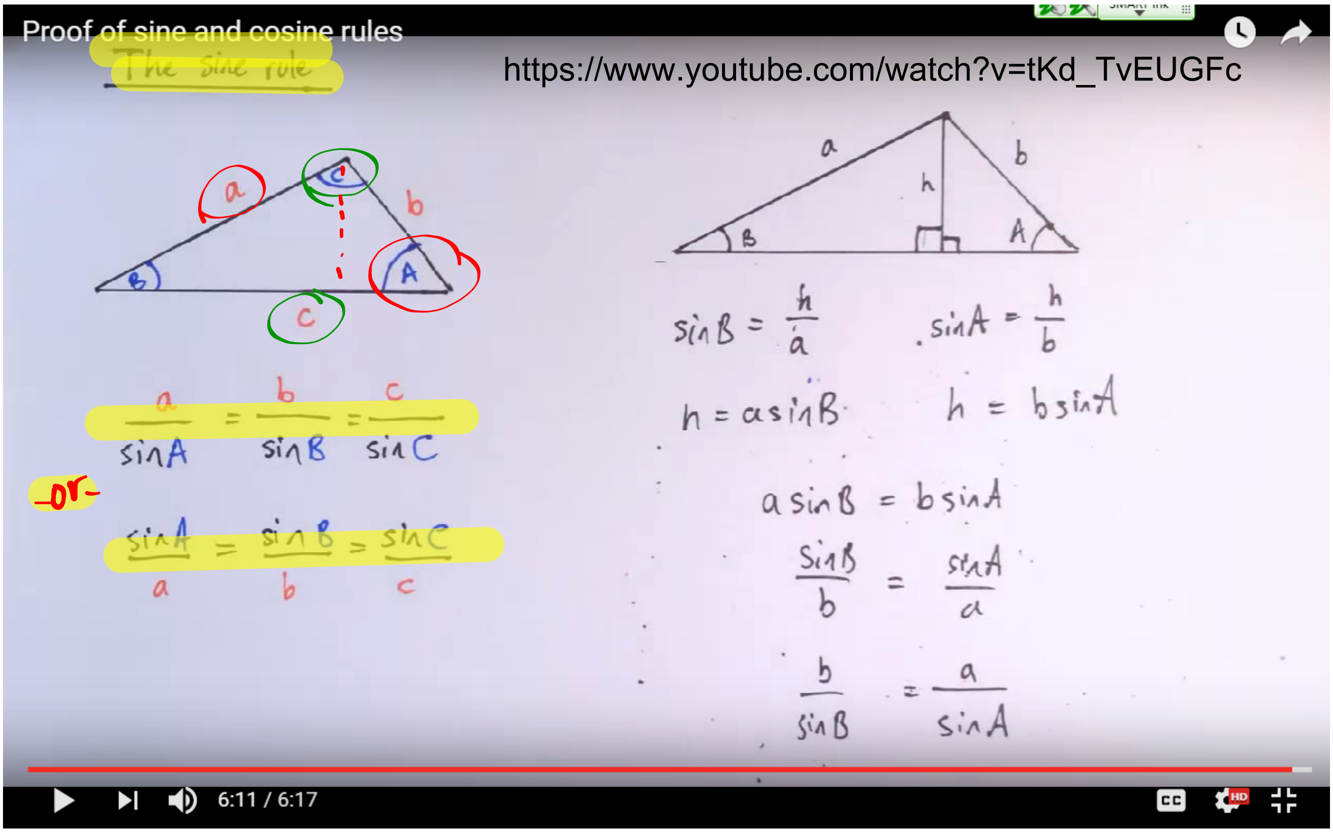
The Pythagorean theorem makes a claim about the relationship between the areas of the three squares drawn on the sides of a right triangle: *the sum of the area of the squares on the two legs is equal to the area of the square on the hypotenuse.* We generally state this relationship algebraically as $a^2 + b^2 = c^2$, where it is understood that a and b represent the length of the two legs of the right triangle, and c represents the length of the hypotenuse.

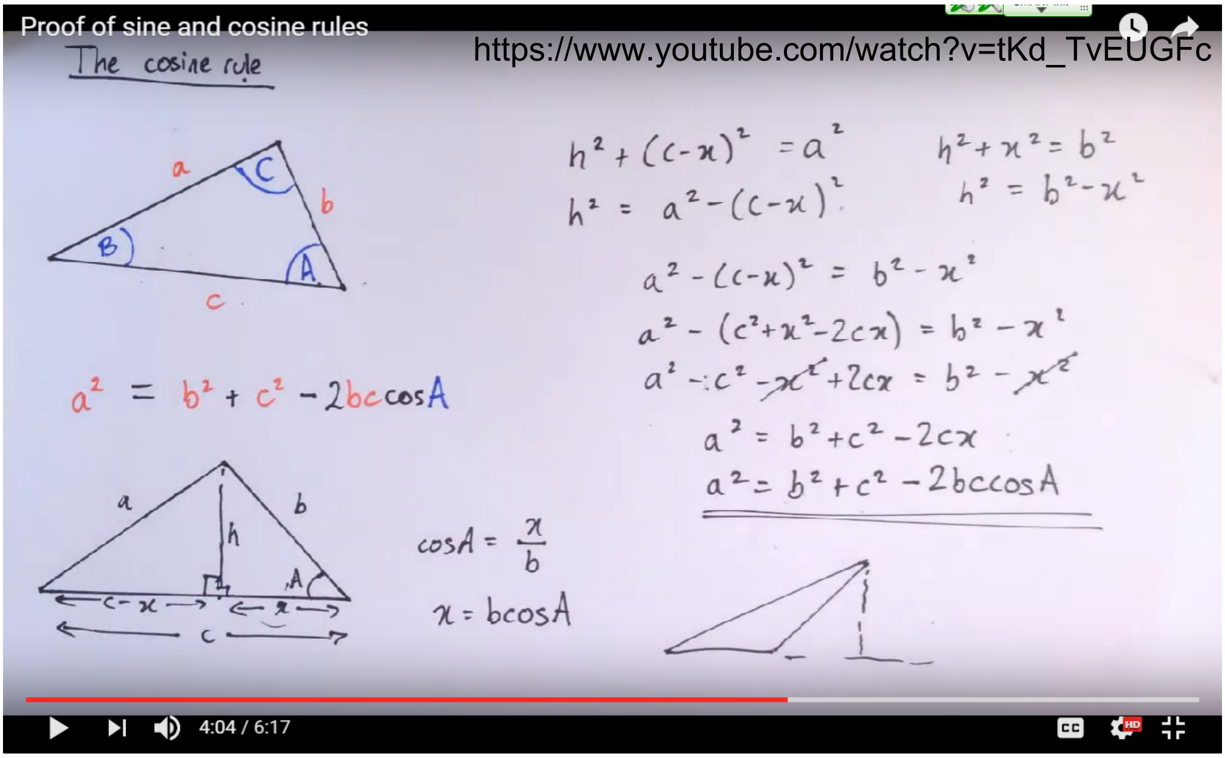
What about non-right triangles? Is there a relationship between the areas of the squares drawn on the sides of a non-right triangle?

Video Notes--> Law of Sine and Cosine Proofs

SKIP

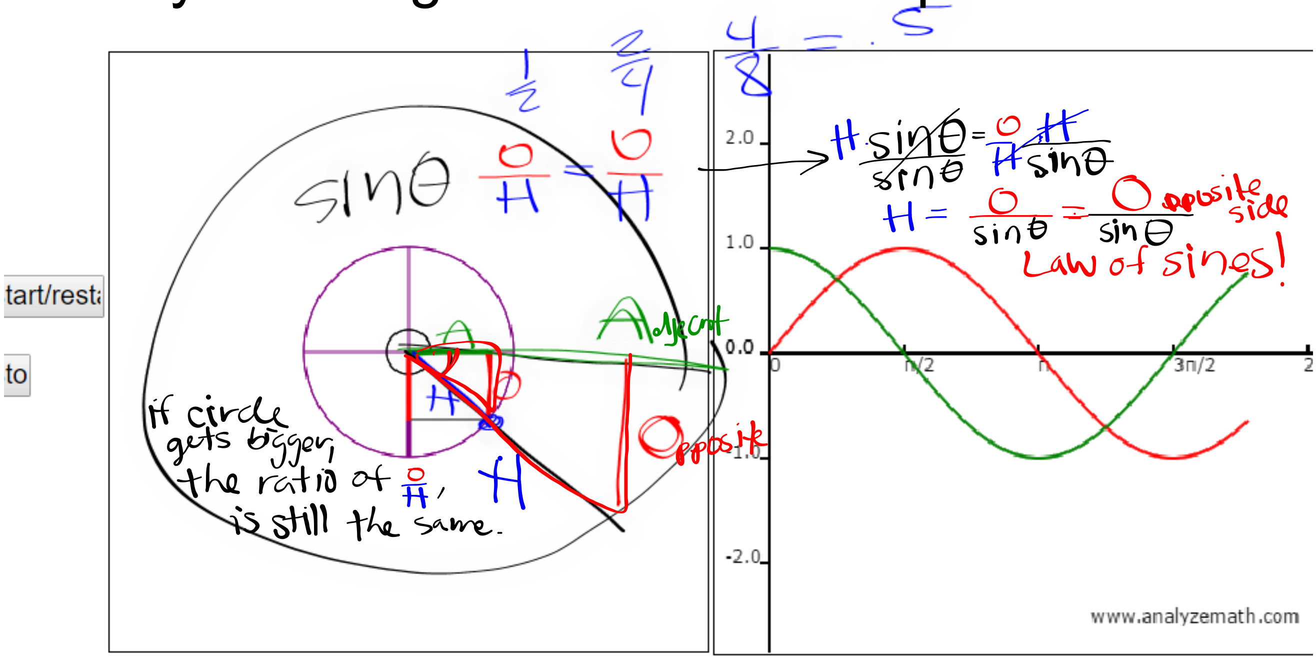



Proof of sine and cosine rules
The sine rule
https://www.youtube.com/watch?v=tKd_TvEUGFc
a/sinA = b/sinB = c/sinC
-or-
sinA/a = sinB/b = sinC/c
sinB = h/a
sinA = h/b
h = a sinB
h = b sinA
a sinB = b sinA
sinB/b = sinA/a
b/sinB = a/sinA
6:11 / 6:17


Proof of sine and cosine rules
https://www.youtube.com/watch?v=tKd_TvEUGFc
The cosine rule
$a^2 = b^2 + c^2 - 2bc\cos A$
$h^2 + (c-x)^2 = a^2$
$h^2 = a^2 - (c-x)^2$
$h^2 + x^2 = b^2$
$h^2 = b^2 - x^2$
$a^2 - (c-x)^2 = b^2 - x^2$
$a^2 - (c^2 + x^2 - 2cx) = b^2 - x^2$
$a^2 - c^2 - \cancel{x^2} + 2cx = b^2 - \cancel{x^2}$
$a^2 = b^2 + c^2 - 2cx$
$a^2 = b^2 + c^2 - 2bc\cos A$
$\cos A = \frac{x}{b}$
$x = b\cos A$
4:04 / 6:17

$\sin(x)$ $\cos(x)$ $\tan(x)$

Pretty amazing! The ratios are equal



5. Since each color is actually a variable representing an area of a rectangle, replace the remaining color in your last equation with the expression that gives the area of the rectangles of that color.

   Write your final equation here:

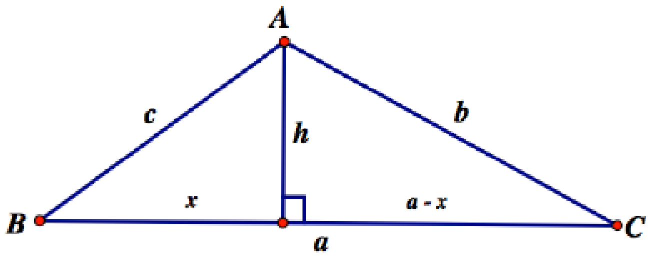
6. Repeat steps 4 and 5 for the other two equations you wrote in step 3. You should end up with three different versions of the **Law of Cosines**, each relating the area of one of the squares drawn on a side of the triangle to the areas of the squares on the other two sides.

These are already written on your assignment

$a^2 =$

$b^2 =$

$c^2 =$



7. Use this diagram to derive one of the forms of the Law of Cosines you wrote above. (Hint: As in the previous task, *More Than Right*, the length of the altitude can be represented in two different ways, both using the Pythagorean theorem and the portions of side *a* that form the legs of two different right triangles.)

8. Use the same diagram above to derive the **Law of Sines**. (Hint: How can you represent the length of the altitude in two different ways using sides *a*, *b*, or *c* and right triangle trigonometry instead of the Pythagorean theorem?)

Name

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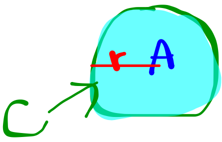
Ready, Set, Go!



Ready

$C = d\pi = (2r)\pi$ $\quad \pi r^2 = A$

Topic: Circumference and area

Use the given information to find the indicated value. Leave π in your answers. Include the correct unit.

1. radius = 3 ft

circumference:

area:

2. diameter = 14 cm

circumference:

area:

3. circumference = 38π km

radius:

area:

4. area = 49π in^2

diameter:

circumference:

5. circumference = 15π mi

radius:

area

6. area = 121π m^2

radius:

circumference:

θ = theta

Solve for the missing angle. Round your answers to the nearest degree. 32.1 = 32°

(Hint: In problems 10, 11, and 12, get the trig function alone. Then solve for θ.)

7. $\cos\theta = \frac{1}{6}$ $\cos^{-1}$ $\cos^{-1}$ $\theta = \cos^{-1}\left(\frac{1}{6}\right)$ $\theta =$ ___

8. $\tan\theta = \frac{2}{3}$

9. $\sin\theta = \frac{7}{8}$

10. $5\sin\theta - 2 = 0$ $+2$ $+2$ $/5$ $/5$ $\sin\theta = \frac{2}{5}$ ~~$\sin^{-1}$~~ $\sin^{-1}$

11. $7\cos\theta - 6 = 0$

12. $4\tan\theta - 1 = 0$

Set Topic: The laws of sine and cosine

Law of Sines: If ABC is a triangle with sides a, b, and c, then $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ or it can be written as: $\frac{\sin A}{a} = \frac{\cos B}{b} = \frac{\tan C}{c}$ Ex: $\frac{6}{10} = \frac{60}{100}$	**Law of Cosines:** If ABC is a triangle with sides a, b, and c, then $a^2 = b^2 + c^2 - 2bc\cos A$ $b^2 = a^2 + c^2 - 2ac\cos B$ $c^2 = a^2 + b^2 - 2ab\cos C$

Ex: $\frac{29}{67} = \frac{16}{x}$

$\frac{1072}{29} = \frac{29 \cdot x}{29}$

$36.9 = x$

Use the law of sines to solve each triangle.

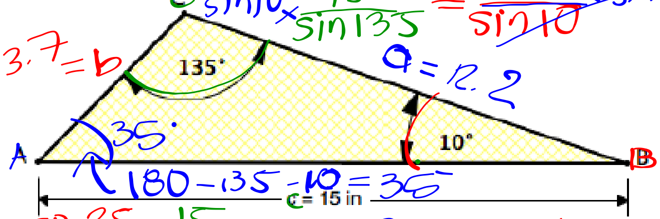
13.

$\frac{c}{\sin c} = \frac{b}{\sin B}$

$\sin 10 \times \frac{15}{\sin 135} = \frac{b}{\sin 10} \times \sin 10$

$3.7 = b$

$a = 12.2$

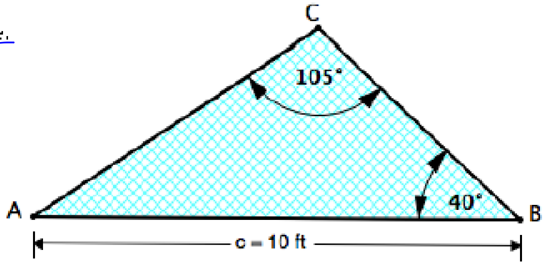


$180 - 135 - 10 = 35$

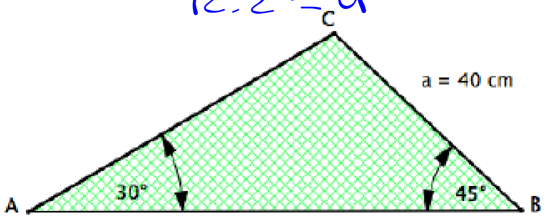
$\sin 35 \times \frac{15}{\sin 135} = \frac{a}{\sin 35} \times \sin 35$

$12.2 = a$

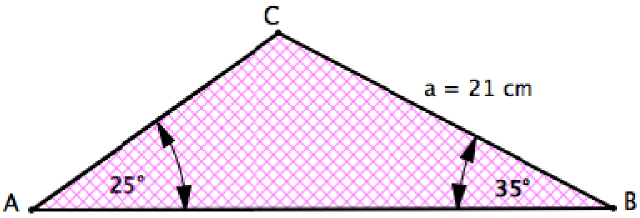
14.



15.



16.



17. What information do you need in order to use the *law of sines*?

you need $\frac{\text{side}}{\text{angle}} = \frac{\text{side}}{\text{angle}}$ atleast 3 pieces

18. Use the *law of cosines* to find the remaining angles and side of the triangle.

$180 - 115 - 40$

$C = 25$

$a^2 = b^2 + c^2 - 2b \cdot c \cos A$

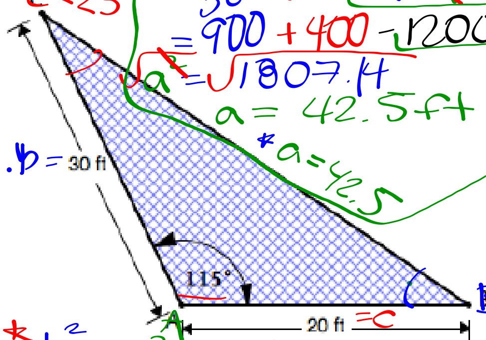
$= 30^2 + 20^2 - 2(30)(20)\cos 115$

$= 900 + 400 - 1200\cos 115$

$\sqrt{a^2} = \sqrt{1807.14}$

$a = 42.5\text{ ft}$

$*a = 42.5$



$*b^2 = a^2 + c^2 - 2a \cdot c \cos B$

$30^2 = 42.5^2 + 20^2 - 2(42.5)(20)\cos B$

$900 = 2206.25 - 1700 \cdot \cos B$

$-2206.25 \quad -2206.25$

$-1306.25 = -1700 \cdot \cos B$

$\overline{-1700} \quad \overline{-1700}$

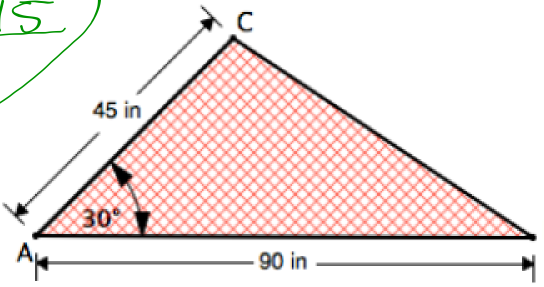
$.768 = \cos B$

$\cos^{-1} \quad \cos^{-1}$

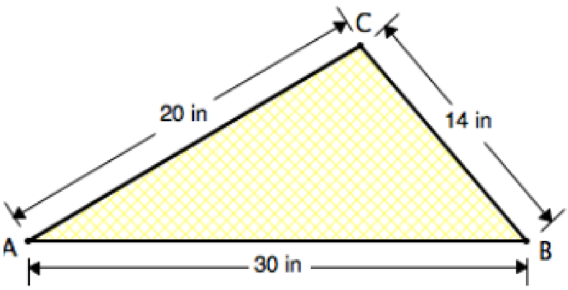
$B \approx 39.8°$

$B \approx 40$

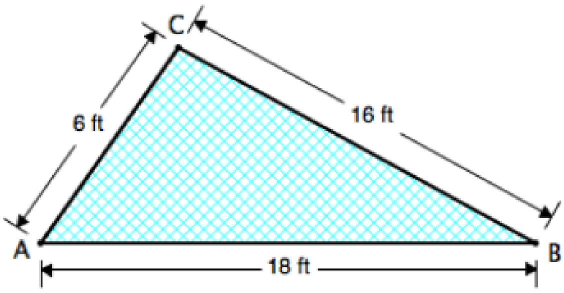
19. Use the *law of cosines* to find the remaining angles and side of the triangle.



20. Use the *law of cosines* to find the three angles of the triangle.



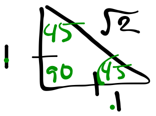
21. Use the *law of cosines* to find the three angles of the triangle.



22. What information do you need in order to use the *law of cosines* to solve a triangle?

want a then I need b & c and m<A
b a c m<B
c a b m<C

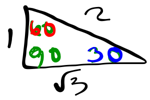
To find a missing side, you need the 2 other sides and opposite angle.



$\sin 45 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\cos 45 = \frac{\sqrt{2}}{2}$

$\tan 45 = \frac{1}{1} = 1$



$\sin 30 = \frac{1}{2}$

$\cos 30 = \frac{\sqrt{3}}{2}$

$\tan 30 = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$

$\sin 60 = \frac{\sqrt{3}}{2}$

* $\cos 60 = \frac{1}{2}$

$\tan 60 = \frac{\sqrt{3}}{1} = \sqrt{3}$

"improper" to leave $\sqrt{\ }$ in denominator.
Multiply it out. top & bottom.

Go

Topic: the trig ratios of the special triangles

What angle would create this fraction/#?

Fill in the missing angle. Do not use a calculator.

23. $\sin\theta = \frac{\sqrt{2}}{2}$	24. $\tan\theta = \sqrt{3}$	25. $\cos\theta = \frac{1}{2}$ 2, 60, 30 * $\cos 60 = \frac{1}{2}$ $\theta = 60°$
26. $\sin\theta = \frac{\sqrt{3}}{2}$	27. $\tan\theta = 1$ ~~$\tan^{-1}$~~ $\tan^{-1}(1)$	28. $\tan\theta = \frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$ $\tan 30$
29. $\sin\theta = \frac{1}{2}$	30. $\cos\theta = \frac{\sqrt{2}}{2}$	31. $\cos\theta = \frac{\sqrt{3}}{2}$